Production-Inventory Systems: Modeling, Forecasting and Control

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with special acknowledgments to:
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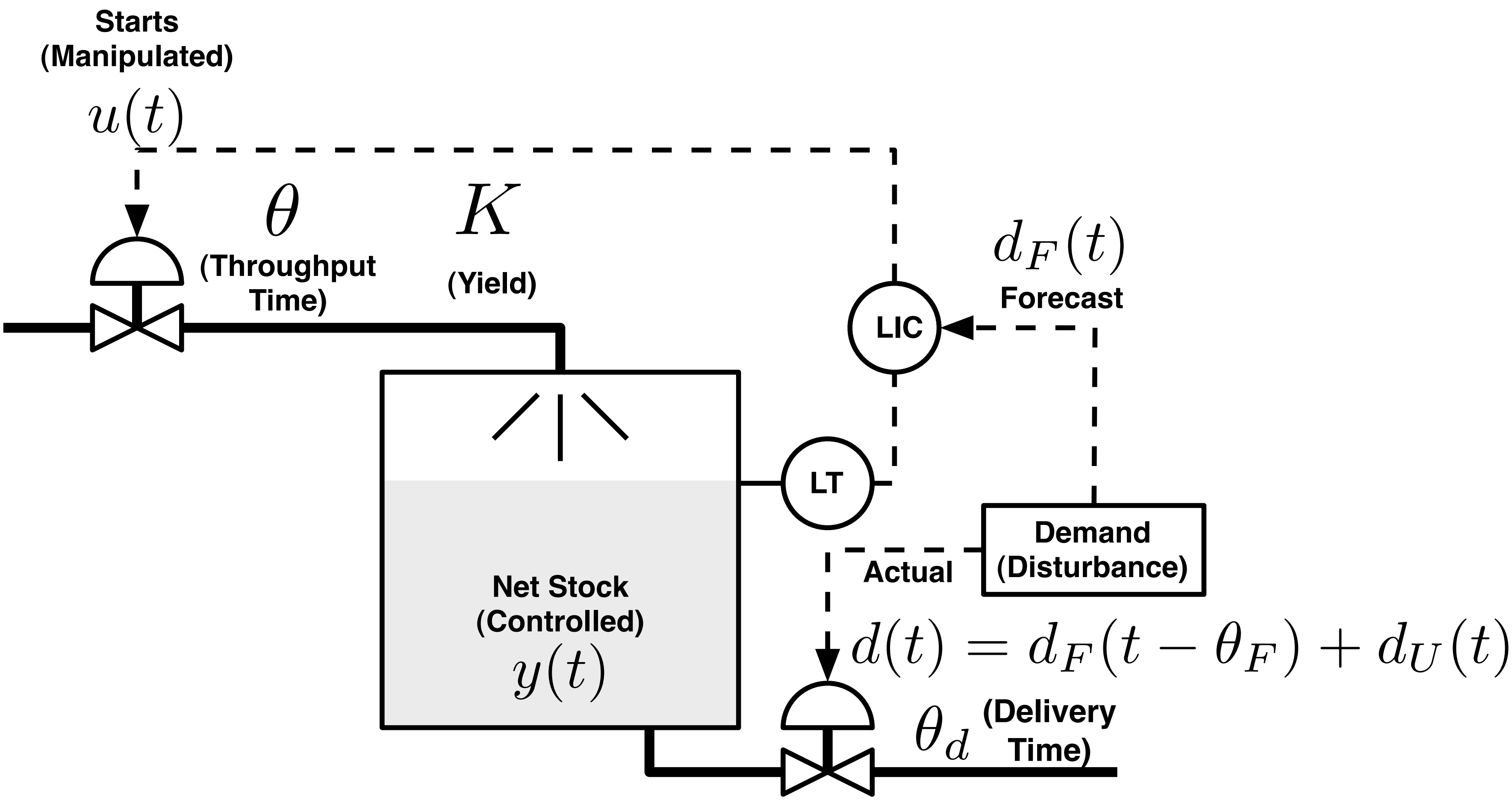
Naresh N. Nandola, ABB

Outline

- Dynamical Model of a Production-Inventory System
- Control Strategies:
  - IMC-PID and 2DoF Feedback-Only IMC
  - 3DoF Combined Feedback/Feedforward IMC
  - Model Predictive Control (MPC)
  - Improved MPC algorithm / Hybrid MPC
- Control-relevant Demand Modeling / Demand Forecasting
- Summary and Conclusions

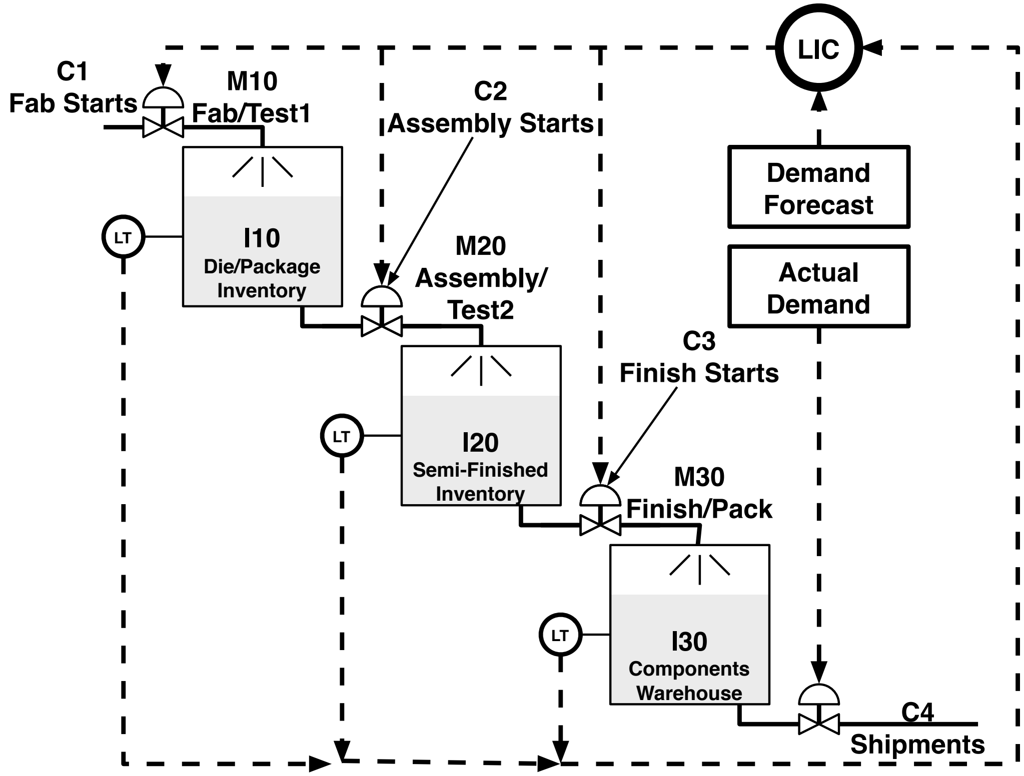
Production-Inventory System



$$y(s) = \frac{Ke^{-\theta s}}{s}u(s) - \frac{e^{-\theta_F s}}{s}d_F(s) - \frac{1}{s}d_U(s)$$

Integrating System with Delays

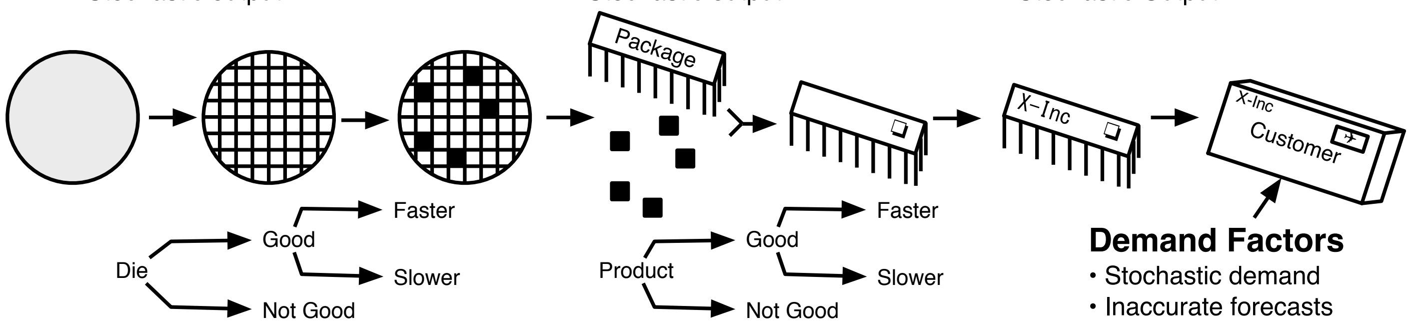
Semiconductor Manufacturing Supply Chain Management



Fabrication/Sort
- Nonlinear Throughput Time (~Weeks)
- Stochastic output

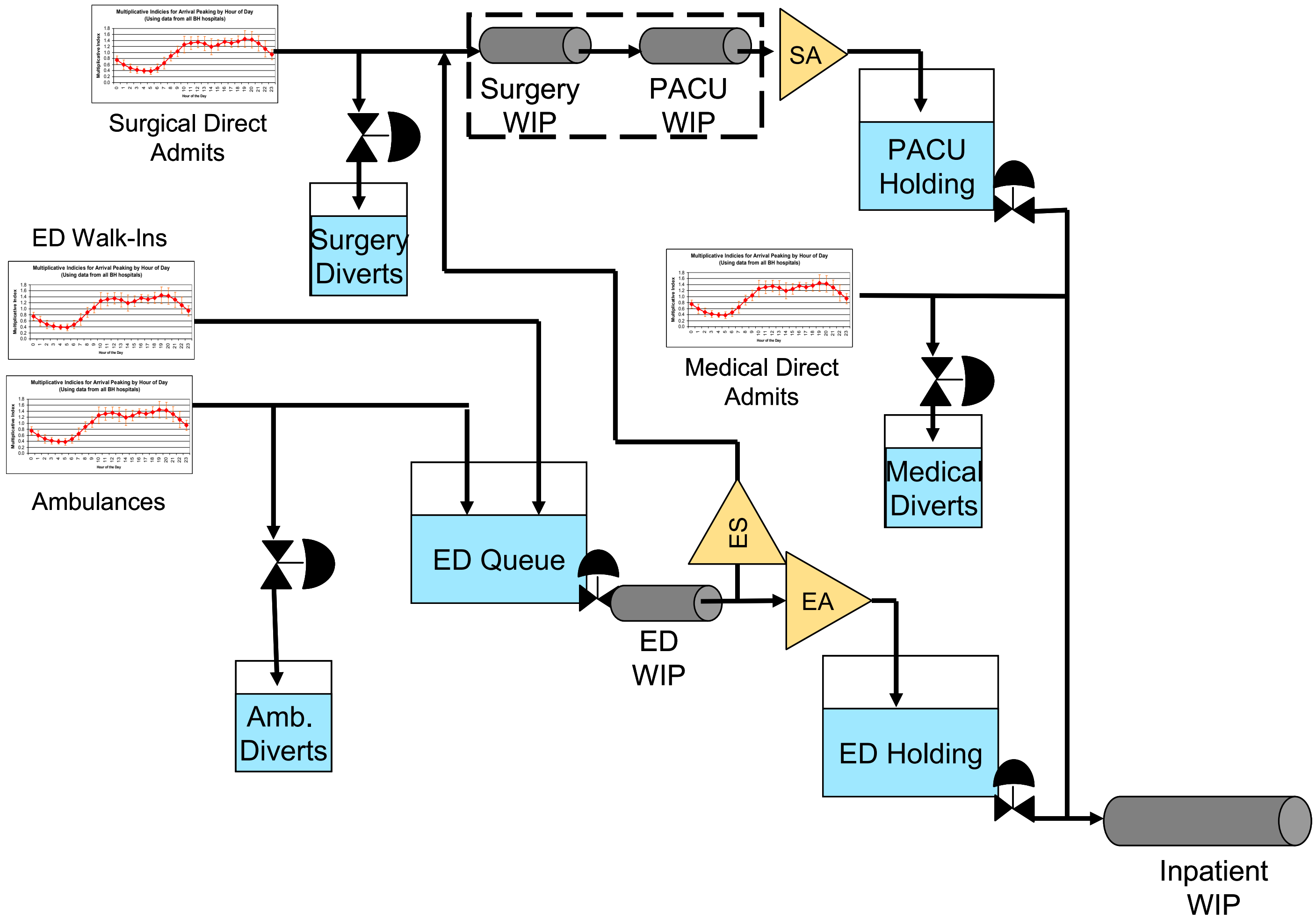
Assembly/Test
- Linear Throughput Time (~Days)
- Stochastic output

Finish/Pack
- Constant Throughput Time (~Shifts)
- Stochastic Output



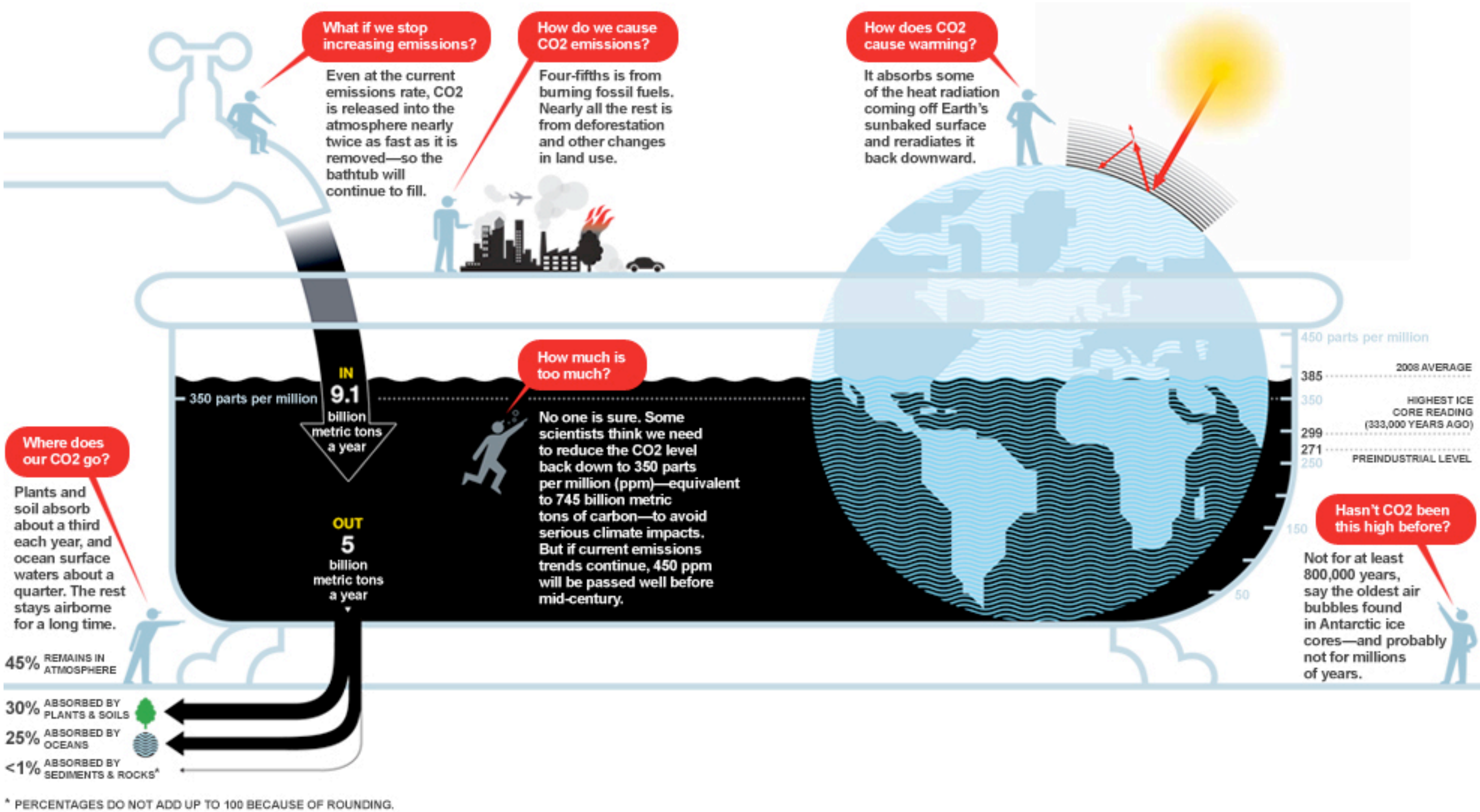
Demand Factors
- Stochastic demand
- Inaccurate forecasts

Whole Hospital Occupancy



- Roche, K.T., D.E. Rivera, and J.K. Cochran, “A control engineering framework for managing whole hospital occupancy,” *Mathematical and Computer Modelling*, Vol. 55, Issues 3-4, pgs. 1401 - 1417, February 2012.

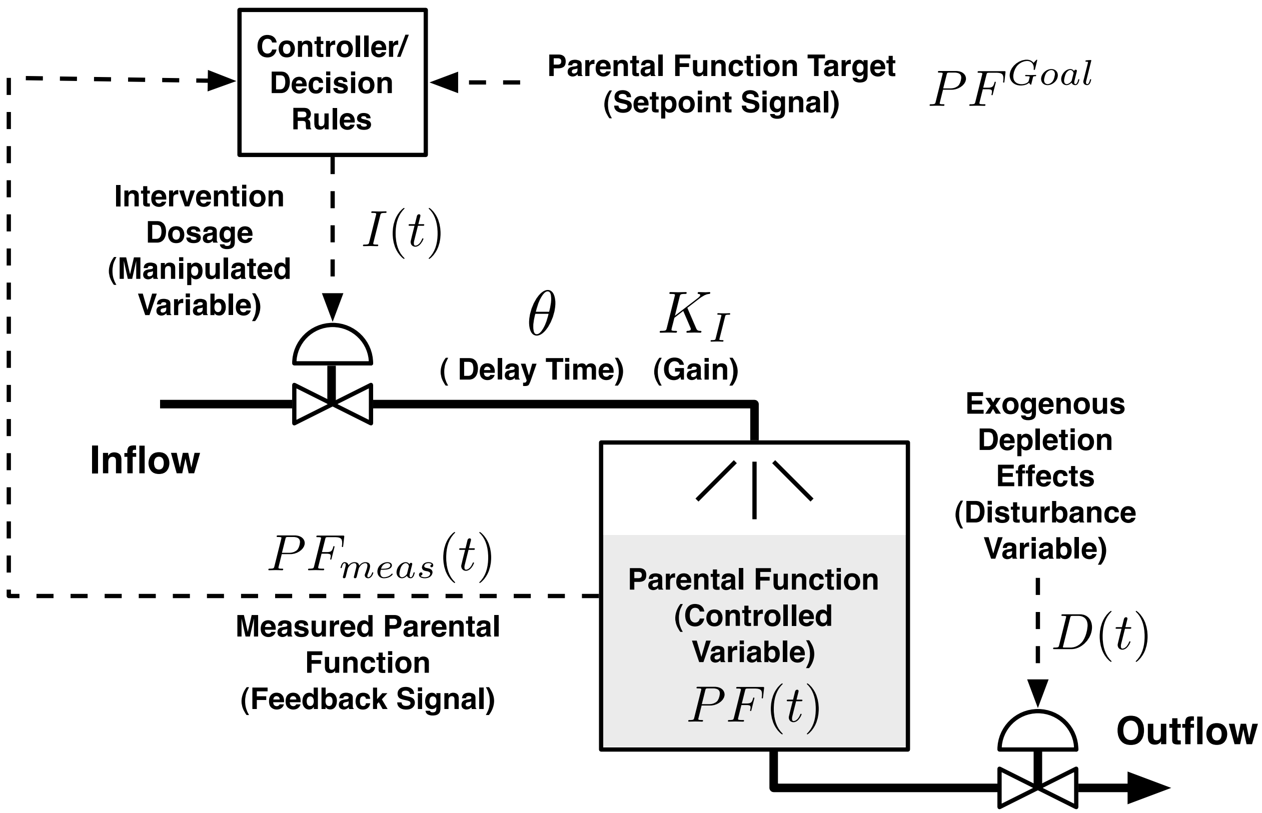
Global Warming/Climate Change



- From *National Geographic Magazine* (http://ngm.nationalgeographic.com/big-idea/05/carbon-bath)

Parental Function-Home Visits Behavioral Intervention as a Production-Inventory Control Problem

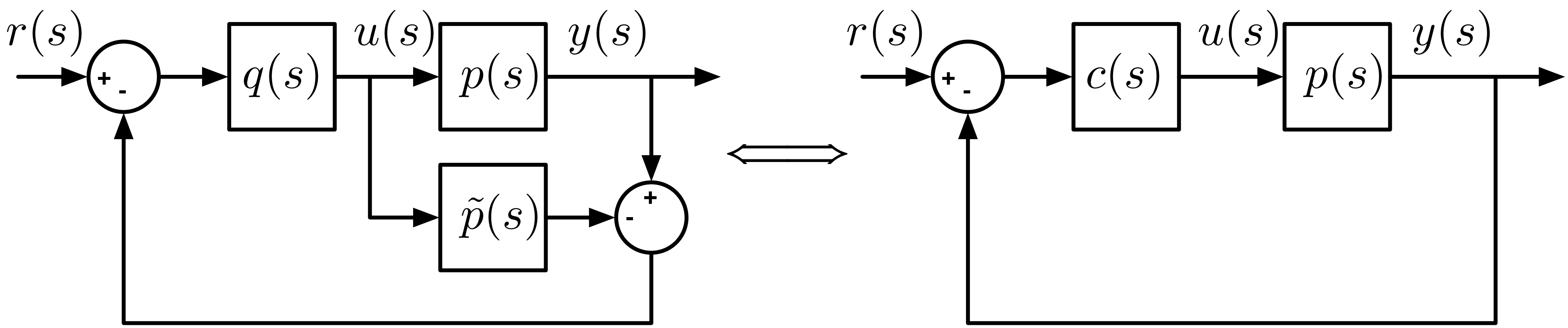
Parental function *PF(t)* is built up by providing an intervention *I(t)* (frequency of home visits), that is potentially subject to delay, and is depleted by potentially multiple disturbances (adding up to *D(t)*).



$$PF(t+1) = PF(t) + K_I\, I(t-\theta) - D(t)$$

- Rivera, D.E., M.D. Pew, and L.M. Collins, "Engineering approaches for the design and analysis of adaptive, time-varying interventions," *Drug and Alcohol Dependence*, Special Issue on Adaptive Treatment Strategies, Vol. 88, Supplement 2, pgs. S31-S40, (2007).

Internal Model Control (IMC) Design Procedure



- Step 1 (Nominal Performance): Obtain an H_2 (ISE)-optimal $q(s)$

  - An external input form is specified (e.g., step or ramp)
  - Closed-form solution for $q(s)$ is obtained
  - Resulting controller is stable and causal

- Step 2 (Robust Stability and Performance)

  - Augment the IMC controller from Step 1 with a filter, $f(s)$.
  - Proper choice and tuning of the filter ensures that:
    the final controller $q(s)$ is proper.
    the control system achieves stability and performance under uncertainty.

IMC-PID Tuning Rules

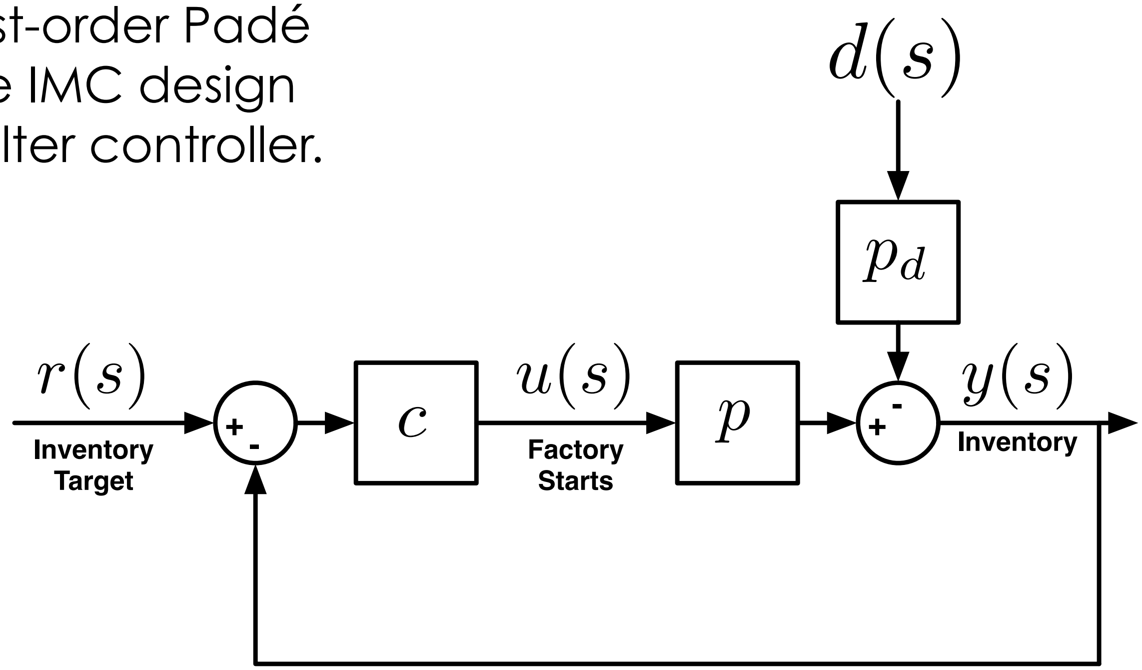
$$p(s) = \frac{Ke^{-\theta s}}{s} \qquad \tilde{p}(s) = \frac{K(-\frac{\theta}{2}s+1)}{s(\frac{\theta}{2}s+1)} \qquad q(s) = \frac{s}{K(\lambda s+1)}$$

Representing the delay with a first-order Padé approximation and applying the IMC design procedure leads to the PID with filter controller.

$$c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)\frac{1}{(\tau_F s + 1)}$$

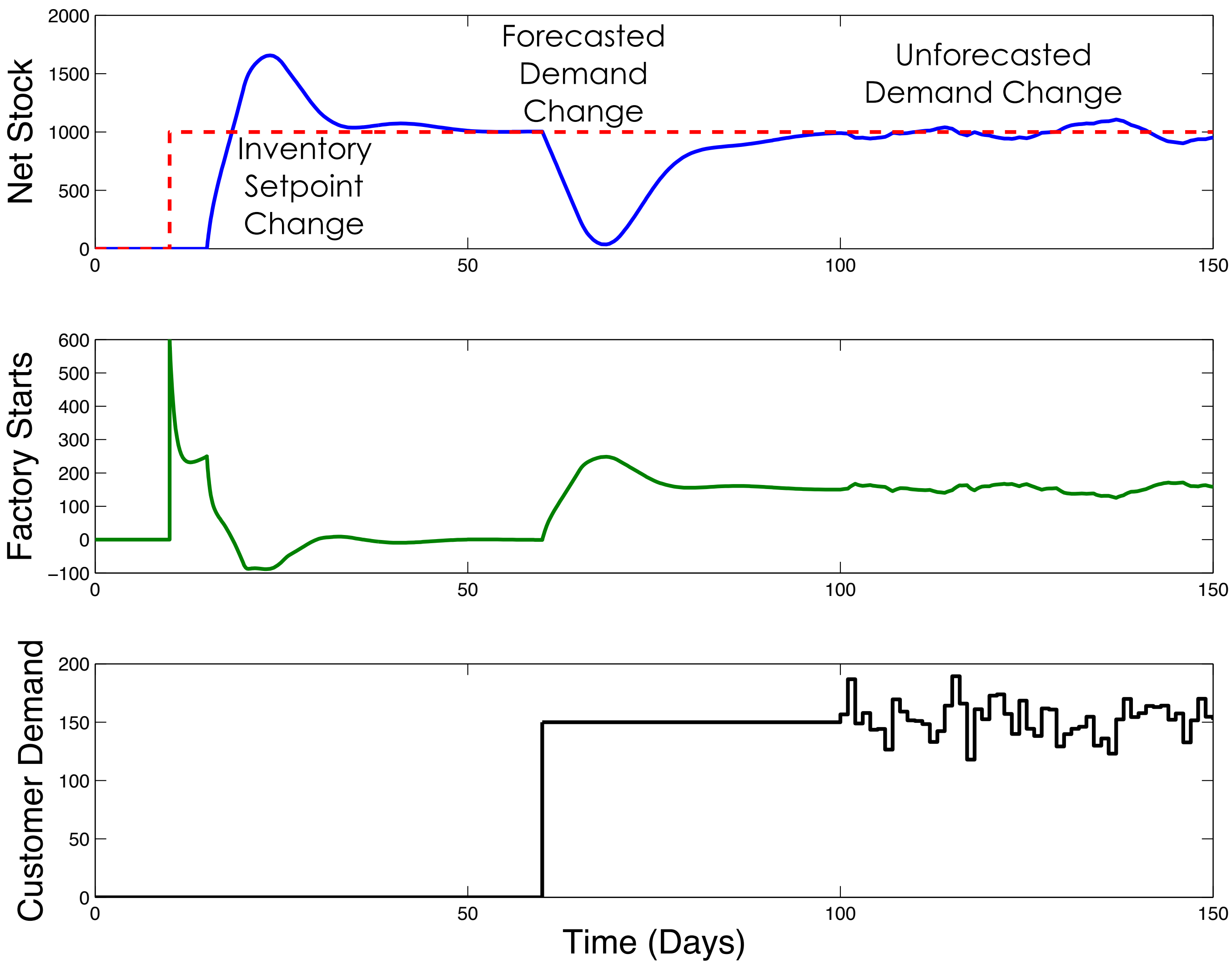
$$K_c = \frac{3\theta + 4\lambda}{K(\theta^2 + 4\theta\lambda + 2\lambda^2)}, \tau_I = \frac{3}{2}\theta + 2\lambda$$

$$\tau_D = \frac{\theta^2 + 2\theta\lambda}{3\theta + 4\lambda}, \tau_F = \frac{\theta\lambda^2}{\theta^2 + 4\theta\lambda + 2\lambda^2}$$



D.E. Rivera, M. Morari, and S. Skogestad. "**Internal Model Control 4: PID Controller Design**". *Ind. Eng. Chem. Process Des. Dev.* **25**, 252-265, **1986**.

IMC-PID Controller Response



$$K = 1 \quad \theta = 5 \quad \theta_d = 0 \quad \lambda = 5$$

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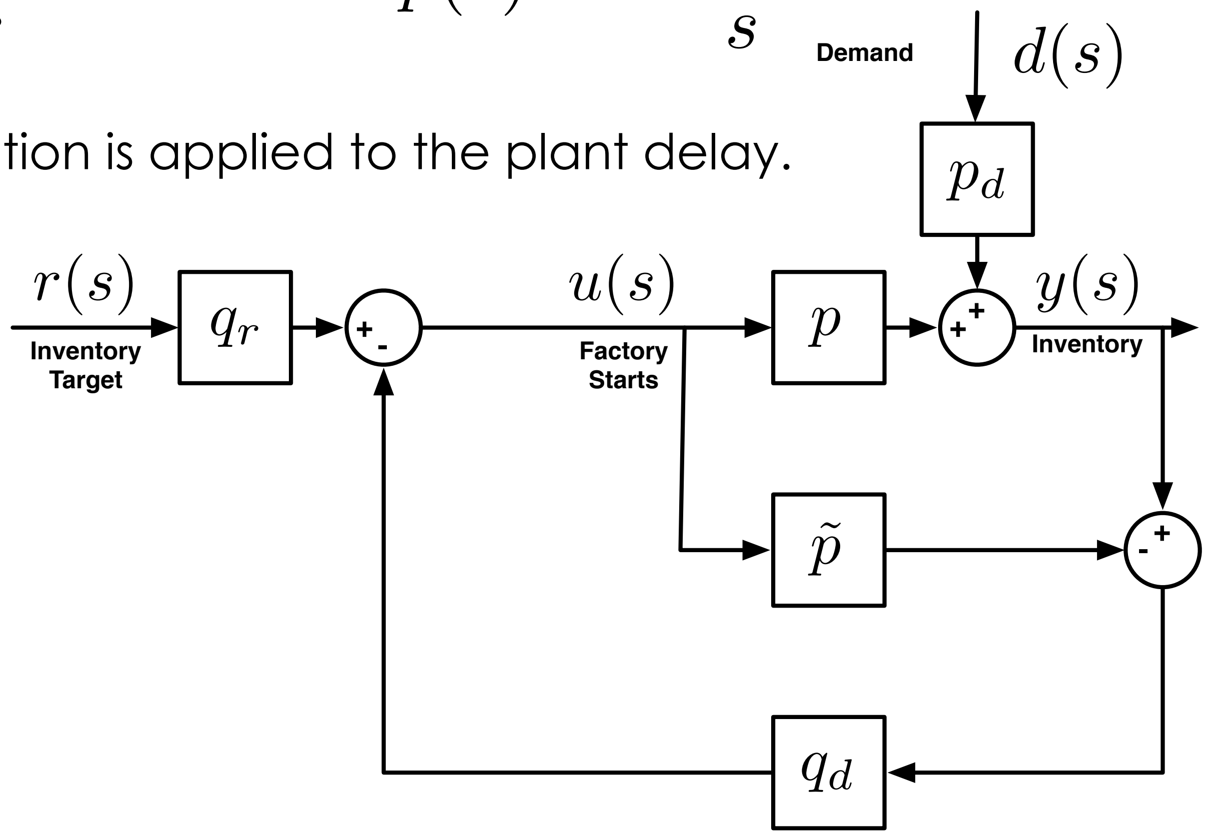
Two Degree-of-Freedom (2DoF) Feedback-Only IMC

$$p(s) = \frac{Ke^{-\theta s}}{s} \qquad \tilde{p}(s) = \frac{Ke^{-\theta s}}{s}$$

No approximation is applied to the plant delay.

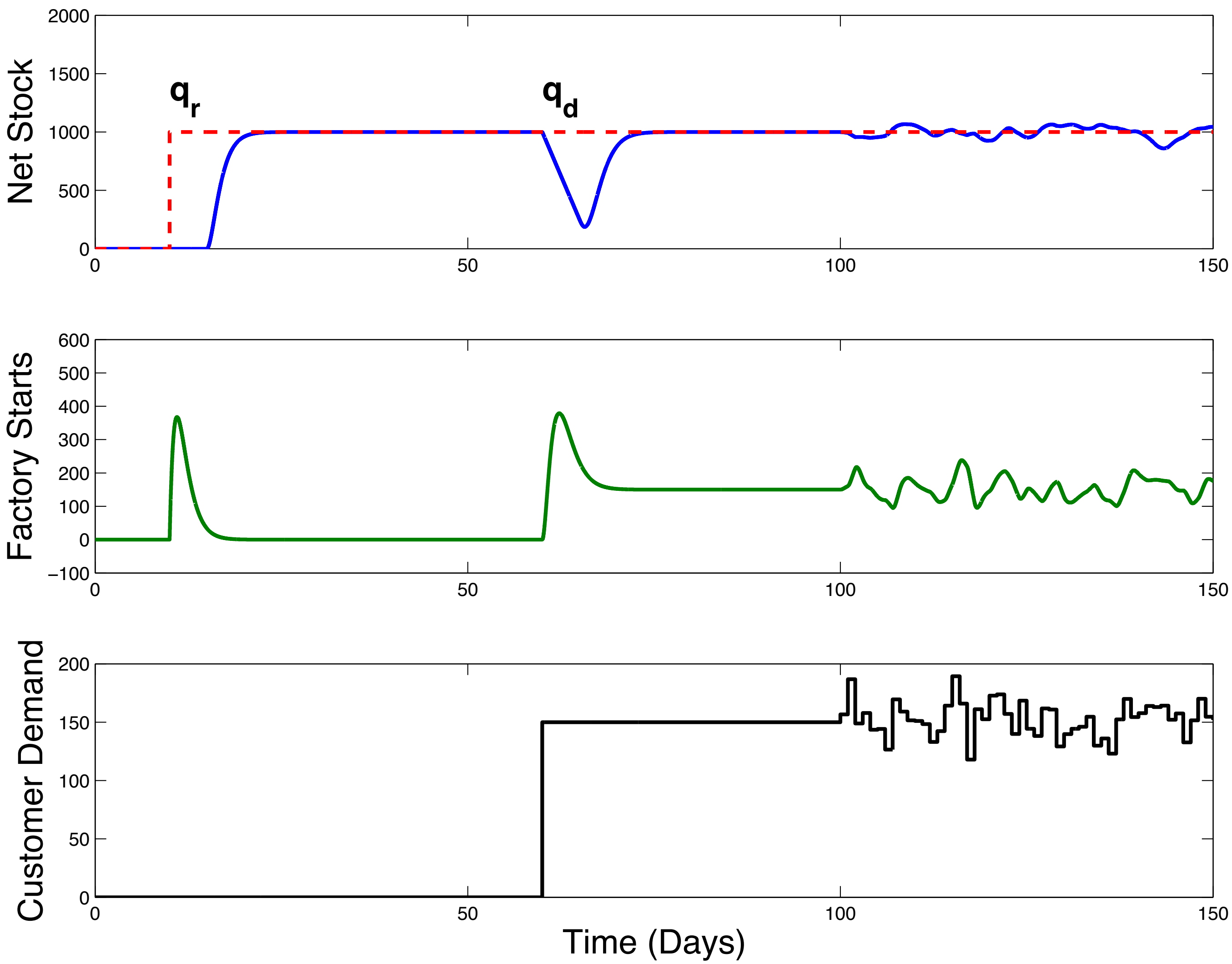
$$q_r(s) = \frac{s}{K} \frac{1}{(\lambda_r s + 1)^{n_r}}$$

$$q_d(s) = \frac{s(\theta s + 1)}{K} \frac{(n_d \lambda_d s + 1)}{(\lambda_d s + 1)^{n_d}}$$



J.D. Schwartz and D.E. Rivera. "**A process control approach to tactical inventory management in production-inventory systems**," *International Journal of Production Economics*, Volume 125, Issue 1, Pages 111-124, 2010.

2DoF Feedback-Only IMC



$$K = 1 \quad \theta = 5 \quad \theta_d = 0 \quad \lambda_r = 1 \quad n_r = 2 \quad \lambda_d = 2 \quad n_d = 3$$

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3DoF Combined Feedback/Feedforward IMC Control

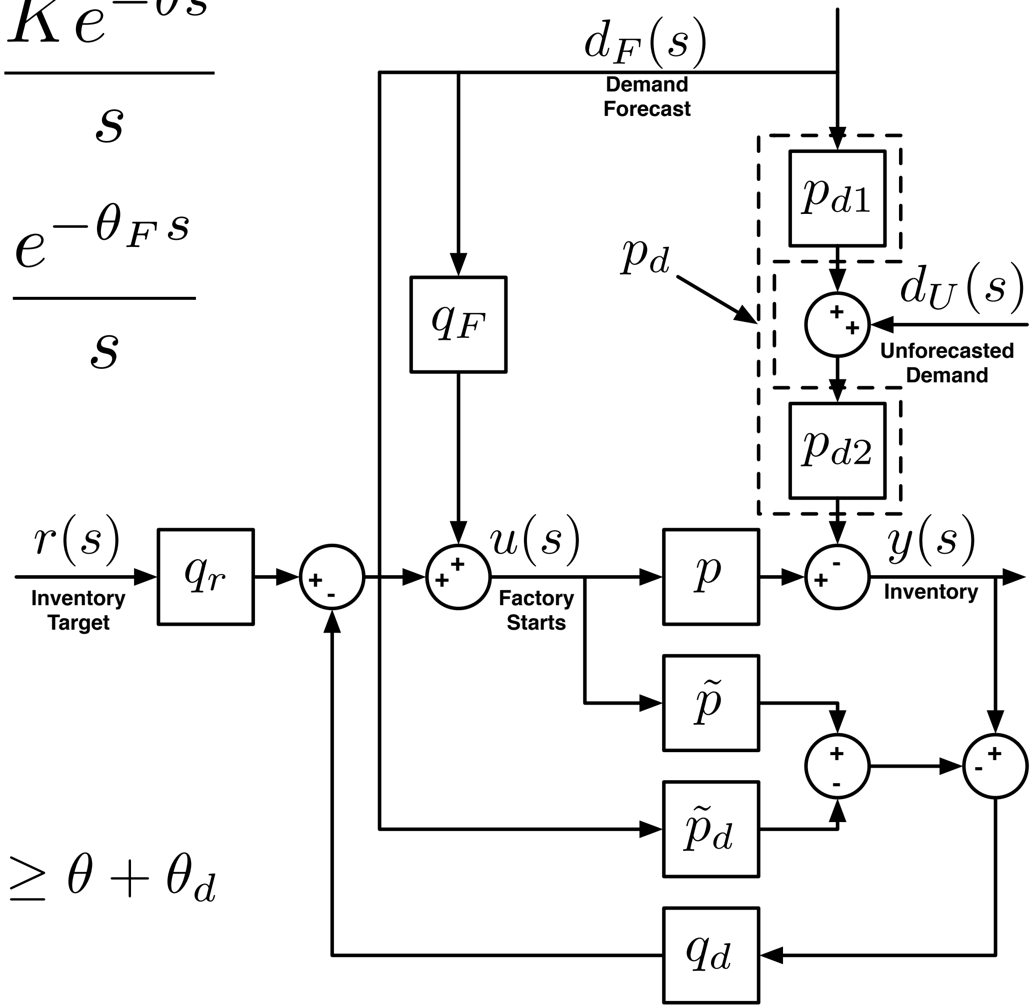
$$p(s) = \frac{Ke^{-\theta s}}{s} \qquad \tilde{p}(s) = \frac{Ke^{-\theta s}}{s}$$

$$p_d(s) = \frac{e^{-\theta_F s}}{s} \qquad \tilde{p}_d(s) = \frac{e^{-\theta_F s}}{s}$$

$$q_r(s) = \frac{s}{K}\,\frac{1}{(\lambda_r s + 1)^{n_r}}$$

$$q_d(s) = \frac{s(\theta s + 1)}{K}\,\frac{(n_d \lambda_d s + 1)}{(\lambda_d s + 1)^{n_d}}$$

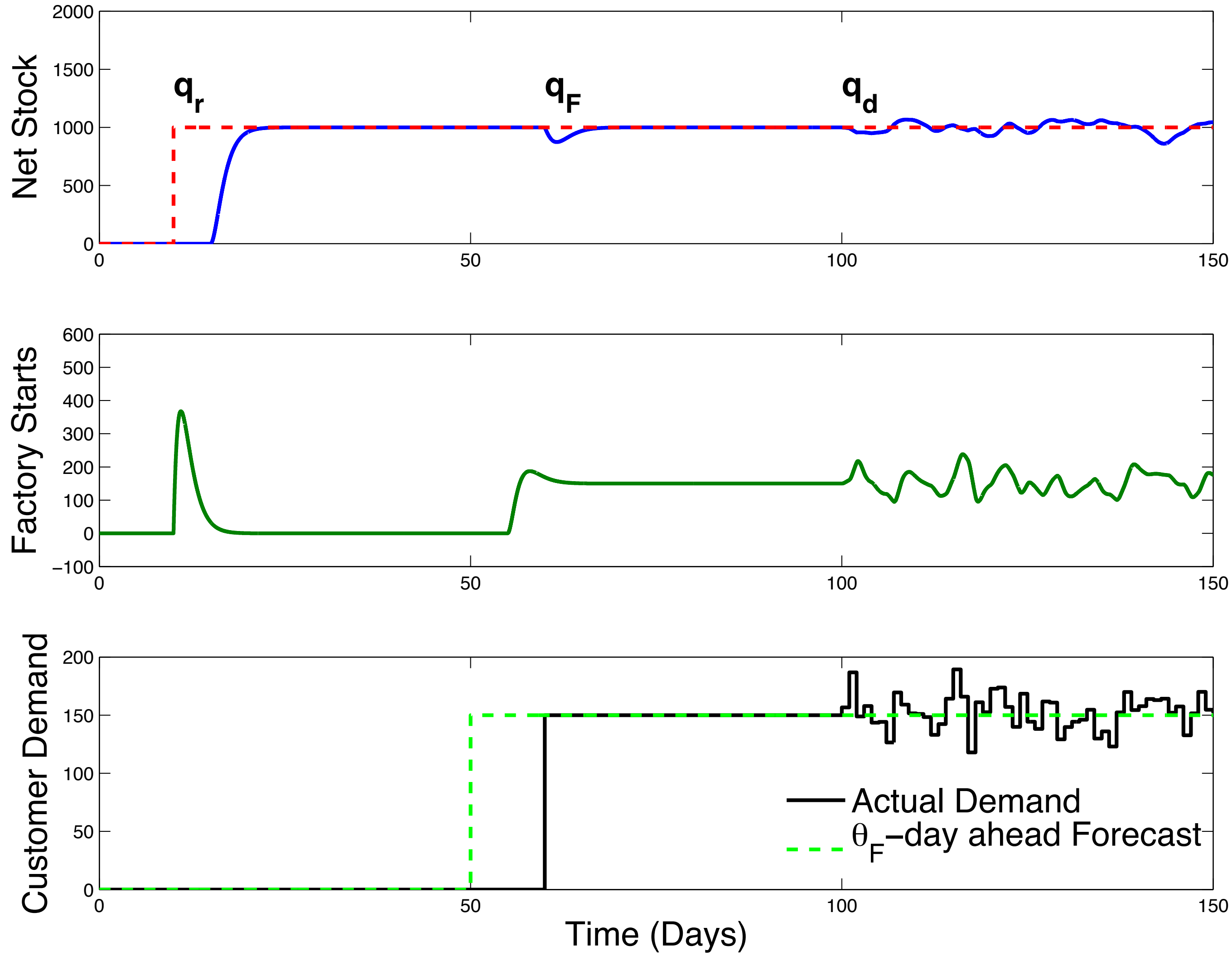
$$q_F(s) = \frac{e^{-(\theta_F - \theta_d - \theta)s}(n_F \lambda_F s + 1)}{K\,(\lambda_F s + 1)^{n_F}}, \qquad \theta_F \geq \theta + \theta_d$$



J.D. Schwartz and D.E. Rivera. "**A process control approach to tactical inventory management in production-inventory systems**," *International Journal of Production Economics*, Volume 125, Issue 1, Pages 111-124, 2010.

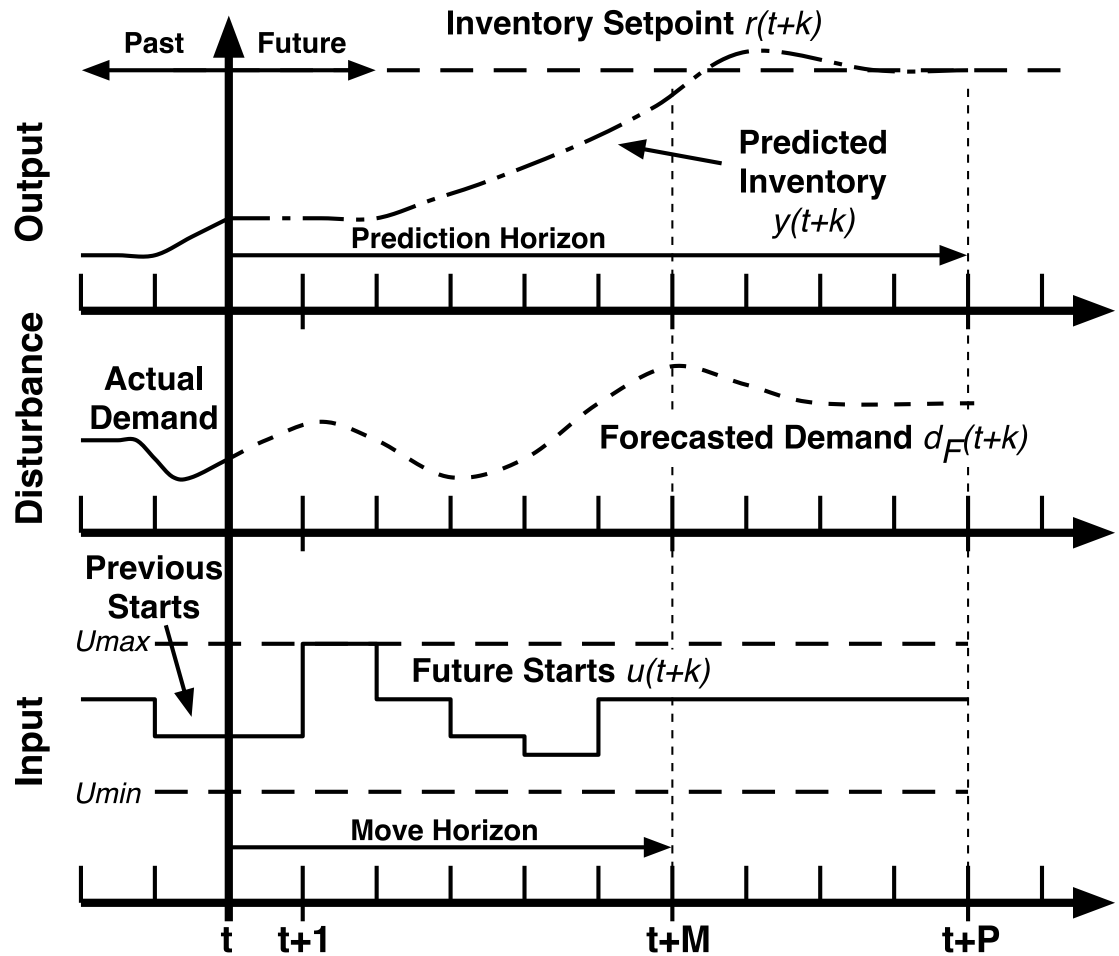
3DoF Combined Feedback/Feedforward IMC Control

$$K = 1 \quad \theta = 5 \quad \theta_d = 0 \quad \theta_F = 10$$



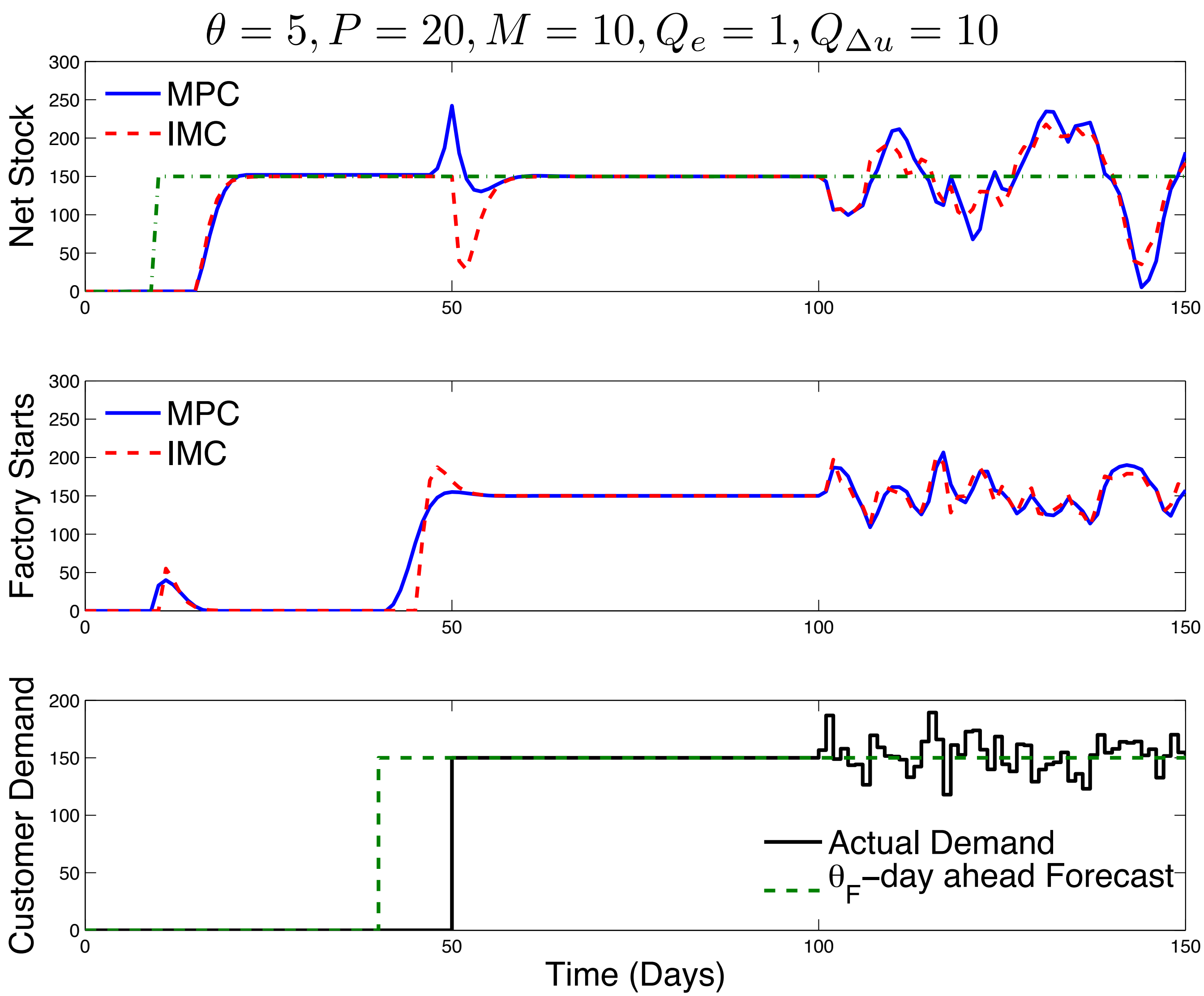
$$\lambda_r = 1 \quad n_r = 2 \quad \lambda_F = 1 \quad n_F = 3 \quad \lambda_d = 2 \quad n_d = 3$$

Model Predictive Control (MPC)



$$\min_{\Delta u(k|k)\ldots\Delta u(k+M-1|k)} \underbrace{\sum_{\ell=1}^{P} Q_e(\ell)(\hat{y}(k+\ell|k) - r(k+\ell))^2}_{\text{Keep Inventories at Planning Setpoints}} + \underbrace{\sum_{\ell=1}^{M} Q_{\Delta u}(\ell)(\Delta u(k+\ell-1|k))^2}_{\text{Penalize Changes in Factory Starts}}$$

IMC/MPC Comparison



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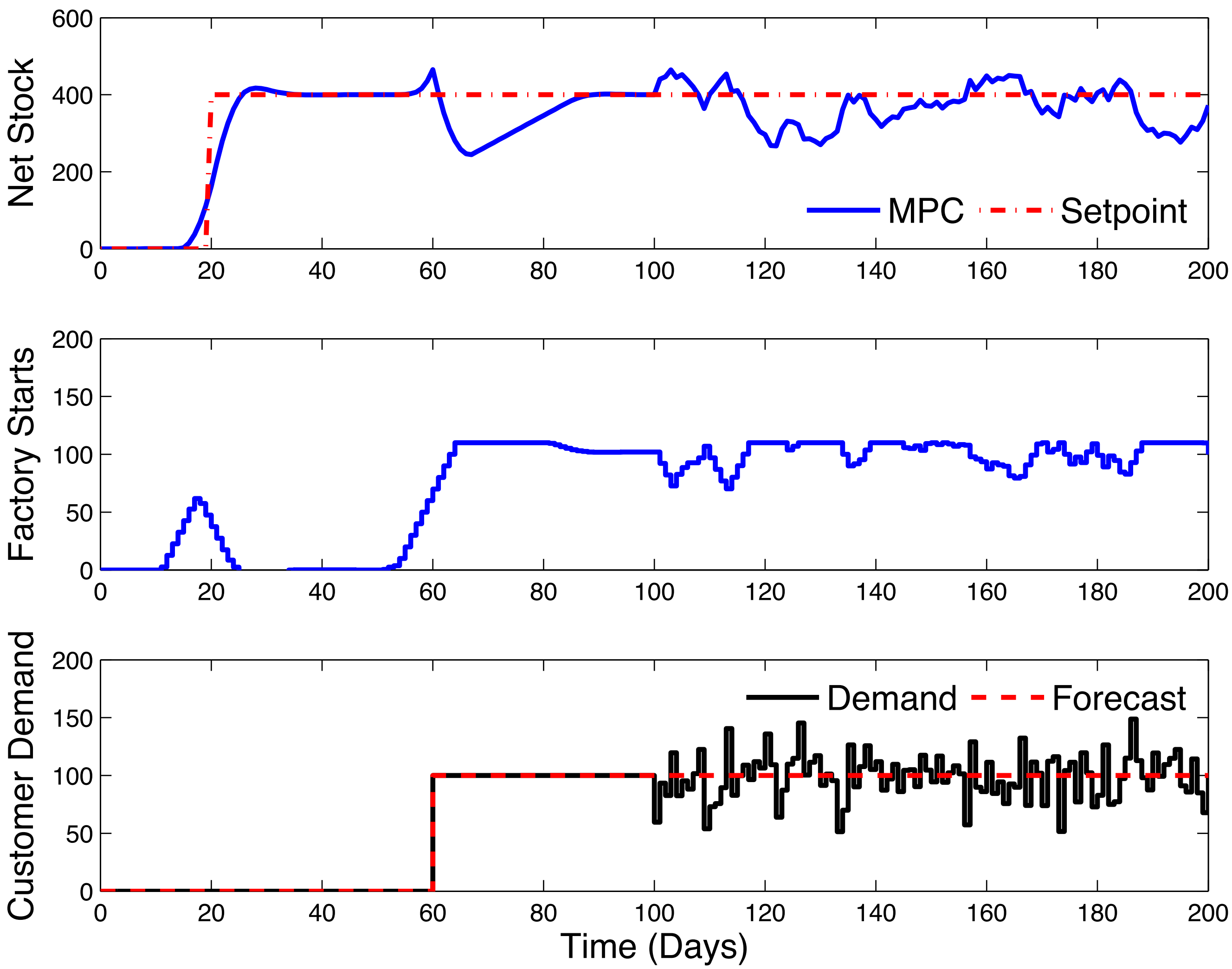
Constrained MPC (with Stpt Anticipation)



Simulation under conditions of active constraints in net stock and factory starts.

Some Observations

- Feedback-only control strategies (even if multi-degree-of-freedom) are unsatisfactory (in general).

- Combined feedback-feedforward strategies that rely on the availability of a demand forecast signal are necessary for good, comprehensive control.

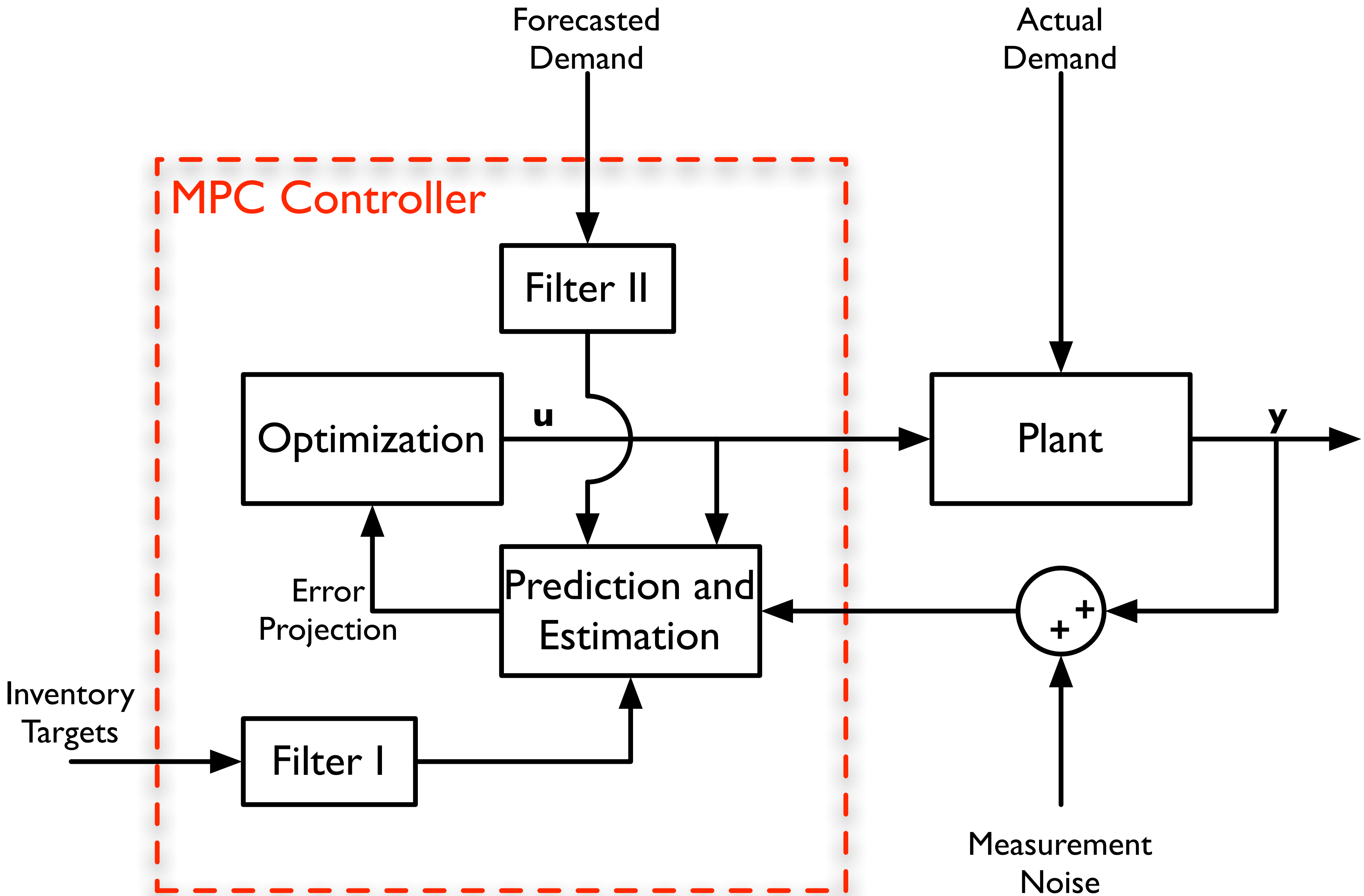
- Model predictive control can provide useful functionality (e.g., constraint handling, anticipation) but the traditional move suppression/single-degree-of-freedom formulation can be lacking.

Motivation for an Improved MPC Formulation

- Integrating dynamics (i.e., ramp responses and disturbances)
- Need to take advantage of anticipated future system inputs (i.e., forecasted demand)
- Multiple degrees-of-freedom (forecasted + unforecasted demand + inventory setpoint tracking) with ease of tuning
- Ability to incorporate problem-specific constraints and possibly hybrid dynamics
- Robustness in the presence of stochasticity and nonlinearity

Nandola, N. and D. E. Rivera, "**An Improved Formulation of Hybrid Model Predictive Control with Application to Production-Inventory Systems**," *IEEE Trans. Control Systems Technology,* Vol. 21, No. 1, pgs. 121-135, 2013.

Block Diagram for 3 DoF MPC Controller



Three Degree-of-Freedom (3-DoF) MPC Tuning

1. Filter I for inventory target setpoint tracking (Type I /asymptotically step signals)

$$f_i(z) = \frac{(1-\alpha_{Ii})z}{z-\alpha_{Ii}}, i = 1, \ldots, n$$

2. Filter II for forecasted demand satisfaction (Type II /asymptotically ramp signals)

$$f_j(z) = \frac{\left[(1-\alpha_{IIj}) + \frac{3}{5}\alpha_{IIj}\right] - \frac{1}{5}\alpha_{IIj}z^{-1} - \frac{2}{5}\alpha_{IIj}z^{-2}}{1-\alpha_{IIj}z^{-1}}, j = 1, \ldots, n$$

Three-degree-of-freedom (3-DoF) MPC tuning (cont.)

- **State estimation and unmeasured disturbance rejection (J.H. Lee and Yu, *Computers and Chemical Engineering*, Vol. 18, No. 1, pgs. 15-37, 1994)**

Step-A1: $X(k|k-1)$: one step ahead prediction using actual measured disturbance (d)

Step-A2: $X(k|k) = X(k|k-1) + K_f(y(k) - \mathcal{C}X(k|k-1))$

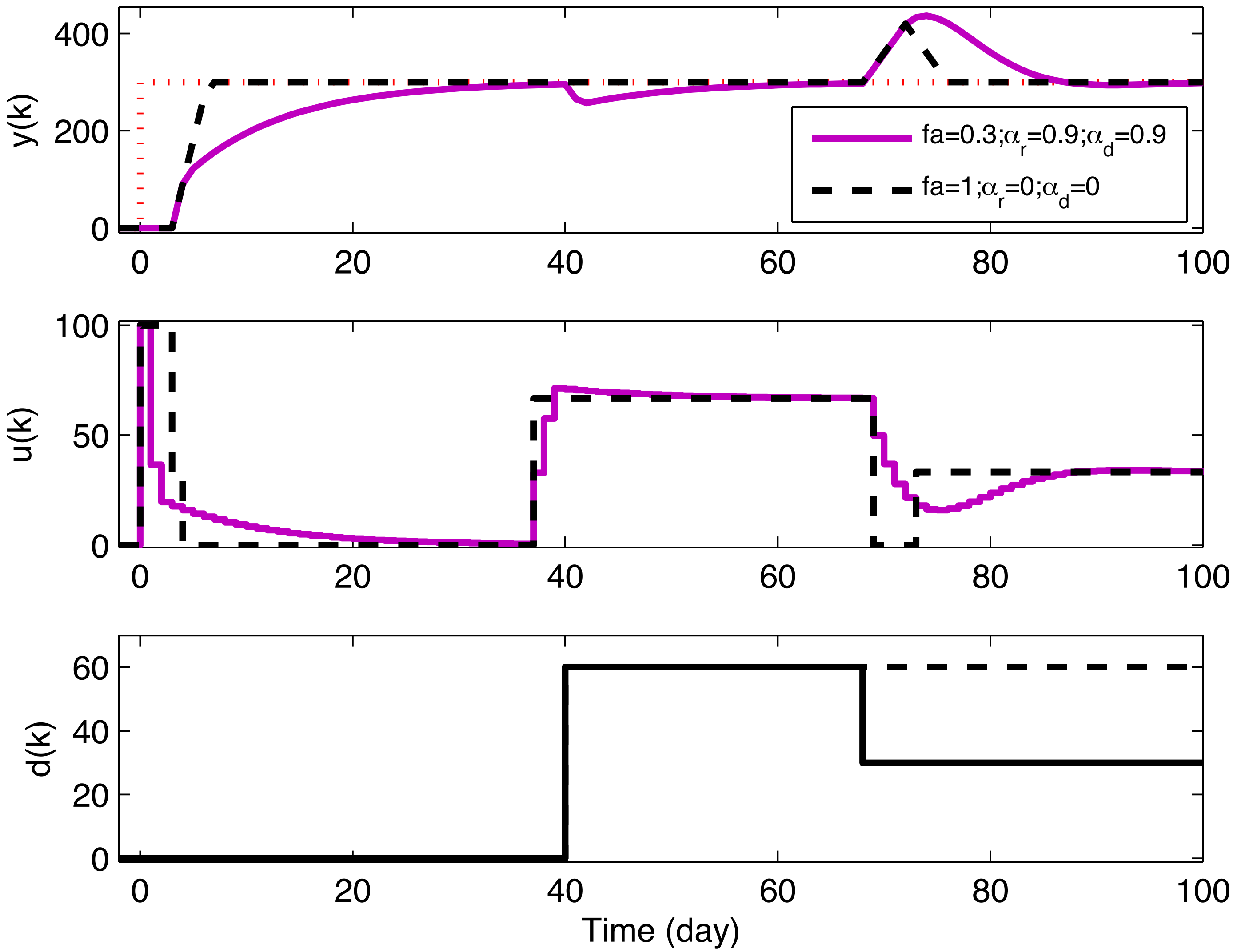
Step-B1: $X_{flt}(k|k-1)$: one step ahead prediction using filtered measured disturbance (d_{flt})

Step-B2: $X_{flt}(k|k) = X_{flt}(k|k-1) + K_f(y(k) - \mathcal{C}X(k|k-1))$

$$\begin{aligned}
K_f &= [0 \quad F_b \quad F_a]^T \\
F_a &= \mathrm{diag}\{(f_a)_1, \cdots, (f_a)_{n_y}\} \\
F_b &= \mathrm{diag}\{(f_b)_1, \cdots, (f_b)_{n_y}\} \\
(f_b)_j &= \frac{{(f_a)_j}^2}{1 + \alpha_j - \alpha_j (f_a)_j}, \quad 0 \le (f_a)_j \le 1, \quad 1 \le j \le n_y
\end{aligned}$$

- $(f_a)_j$ is focused on each output j; constrained to $0 \le (f_a)_j \le 1$
- Speed of dist. rejection is proportional to the tuning parameter $(f_a)_j$

3-DoF MPC for Continuous Input



independent controller adjustment without the need for move suppression!

Controller Model (includes hybrid dynamics)

Plant Model Mixed Logical Dynamical (MLD) Framework

$$
\begin{aligned}
x(k+1) &= Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) + B_dd(k) \\
y(k+1) &= Cx(k+1) + d'(k+1) + \nu(k+1) \\
E_5 &\geq E_2\delta(k) + E_3z(k) - E_4y(k) - E_1u(k) + E_dd(k)
\end{aligned}
$$

d' : Unmeasured disturbance $\quad$ d : Measured disturbance

Disturbance Model

$$
\begin{aligned}
x_w(k+1) &= A_wx_w(k) + B_ww(k) \longrightarrow \text{Integrated white noise} \\
d'(k+1) &= C_wx_w(k+1)
\end{aligned}
$$

$$A_w = diag\{\alpha_1,\ \alpha_1,\ \cdots,\ \alpha_{n_y}\},\ B_w = C_w = I$$

MPC Objective Function

$$\min_{\{[u(k+i)]_{i=0}^{m-1},\ [\delta(k+i)]_{i=0}^{p-1},\ [z(k+i)]_{i=0}^{p-1}\}} J \triangleq \sum_{i=1}^{p} \|(y(k+i)-y_r)\|_{Q_y}^2 + \sum_{i=0}^{m-1} \|(\Delta u(k+i))\|_{Q_{\Delta u}}^2$$

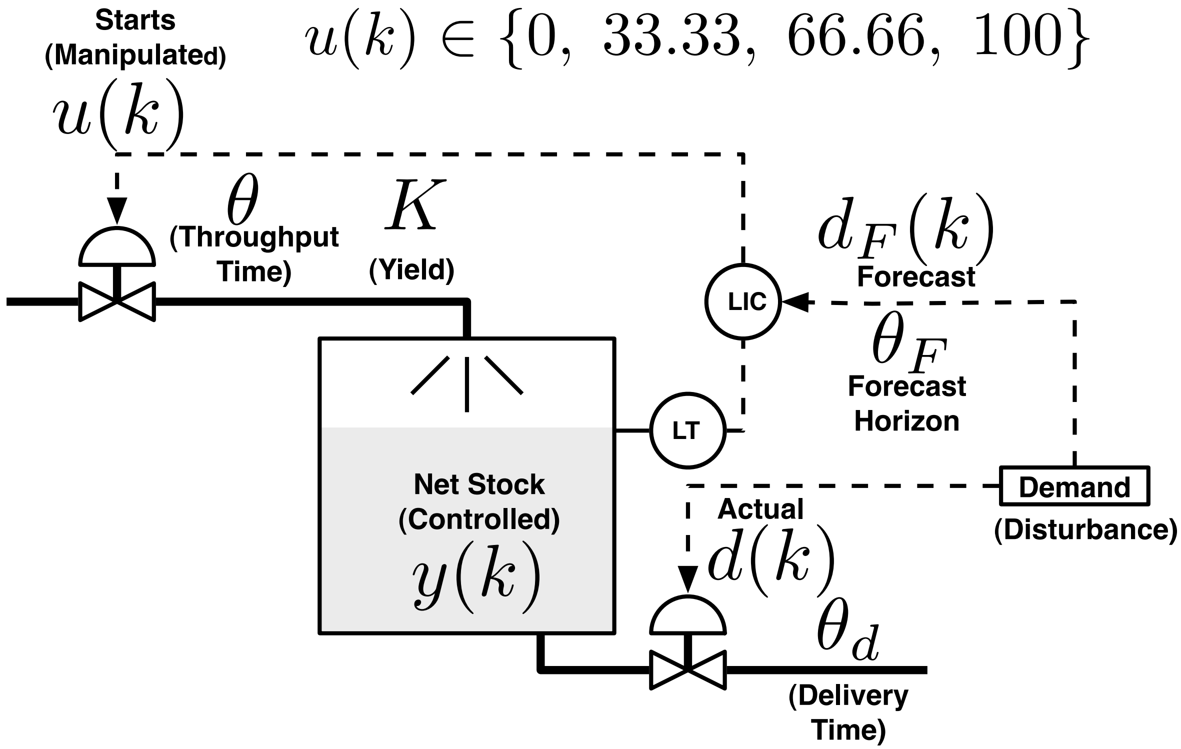
$$+\sum_{i=0}^{m-1} \|(u(k+i)-u_r)\|_{Q_u}^2 + \sum_{i=0}^{p-1} \|(\delta(k+i)-\delta_r)\|_{Q_d}^2 + \sum_{i=0}^{p-1} \|(z(k+i)-z_r)\|_{Q_z}^2$$

Subject to

$$\begin{aligned}
E_5 &\geq E_2\delta(k+i) + E_3 z(k+i) - E_4 y(k+i) - E_1 u(k) + E_d d(k+i),\ 0 \leq i \leq p-1 \\
y_{\min} \leq & \quad y(k+i) \quad \leq y_{\max},\ 1 \leq i \leq p \\
u_{\min} \leq & \quad u(k+i) \quad \leq u_{\max},\ 0 \leq i \leq m-1 \\
\Delta u_{\min} \leq & \quad \Delta u(k+i) \quad \leq \Delta u_{\max},\ 0 \leq i \leq m-1
\end{aligned}$$

Hybrid 3 DoF Model Predictive Control, Production-Inventory System

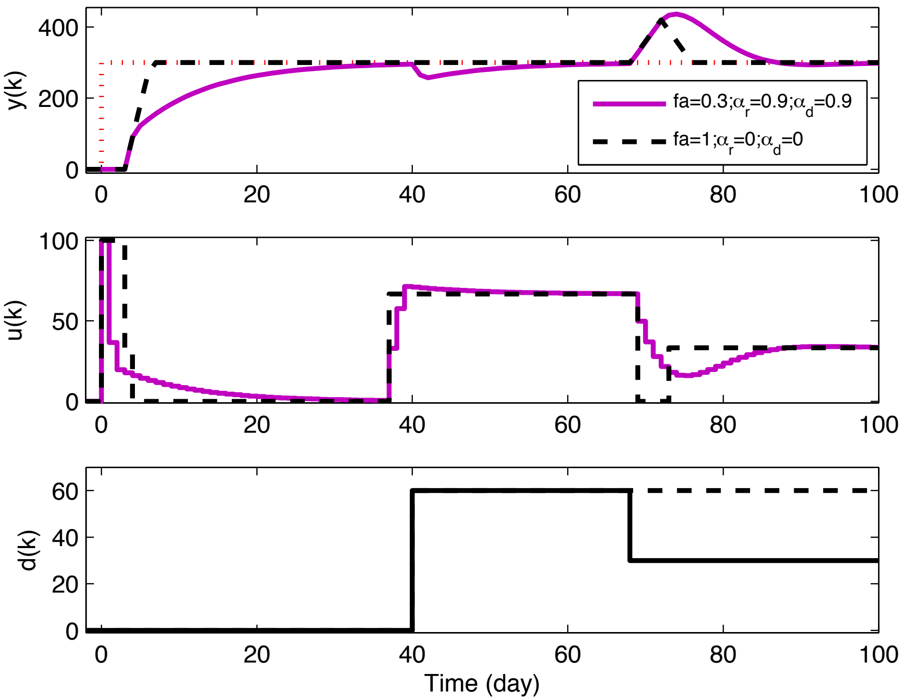
$$u(k) \in \{0,\ 33.33,\ 66.66,\ 100\}$$



$$
\begin{aligned}
y(k+1) &= y(k) + Ku(k-(\theta-1)) - d(k) \\
d(k) &= \underbrace{d_f(k)}_{forecasted} + \underbrace{d_u(k)}_{unforecasted}
\end{aligned}
$$

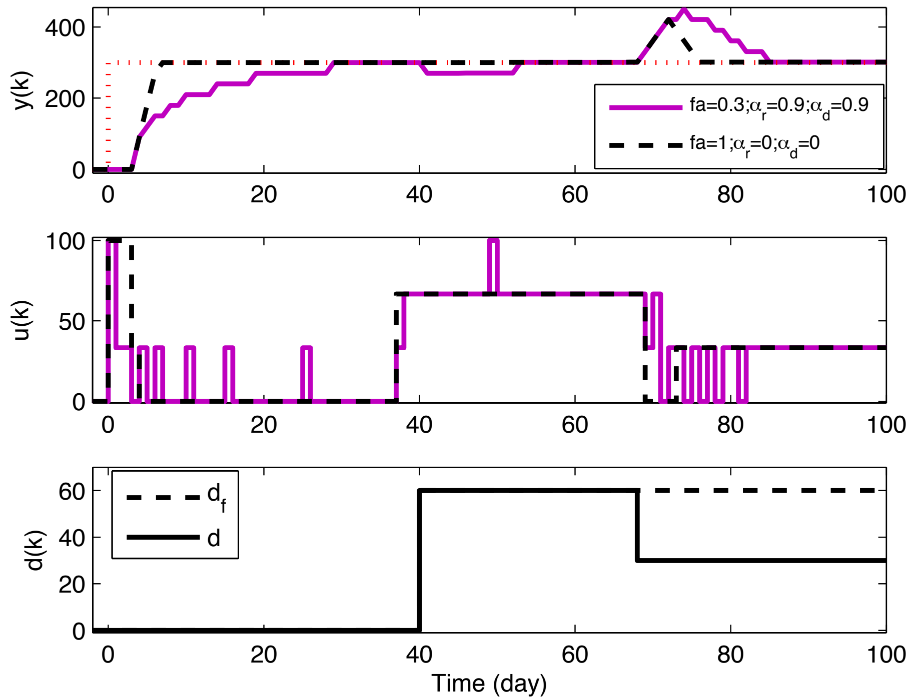
Hybrid vs Continuous 3 DoF MPC
Production-Inventory System

Continuous u(t)



Discrete-level u(t)

$$u(k) \in \{0,\ 33.33,\ 66.66,\ 100\}$$



Solution involves solving a *Mixed Integer Quadratic Program* (MIQP) to address continuous error but discrete-level inputs (i.e., a hybrid problem).

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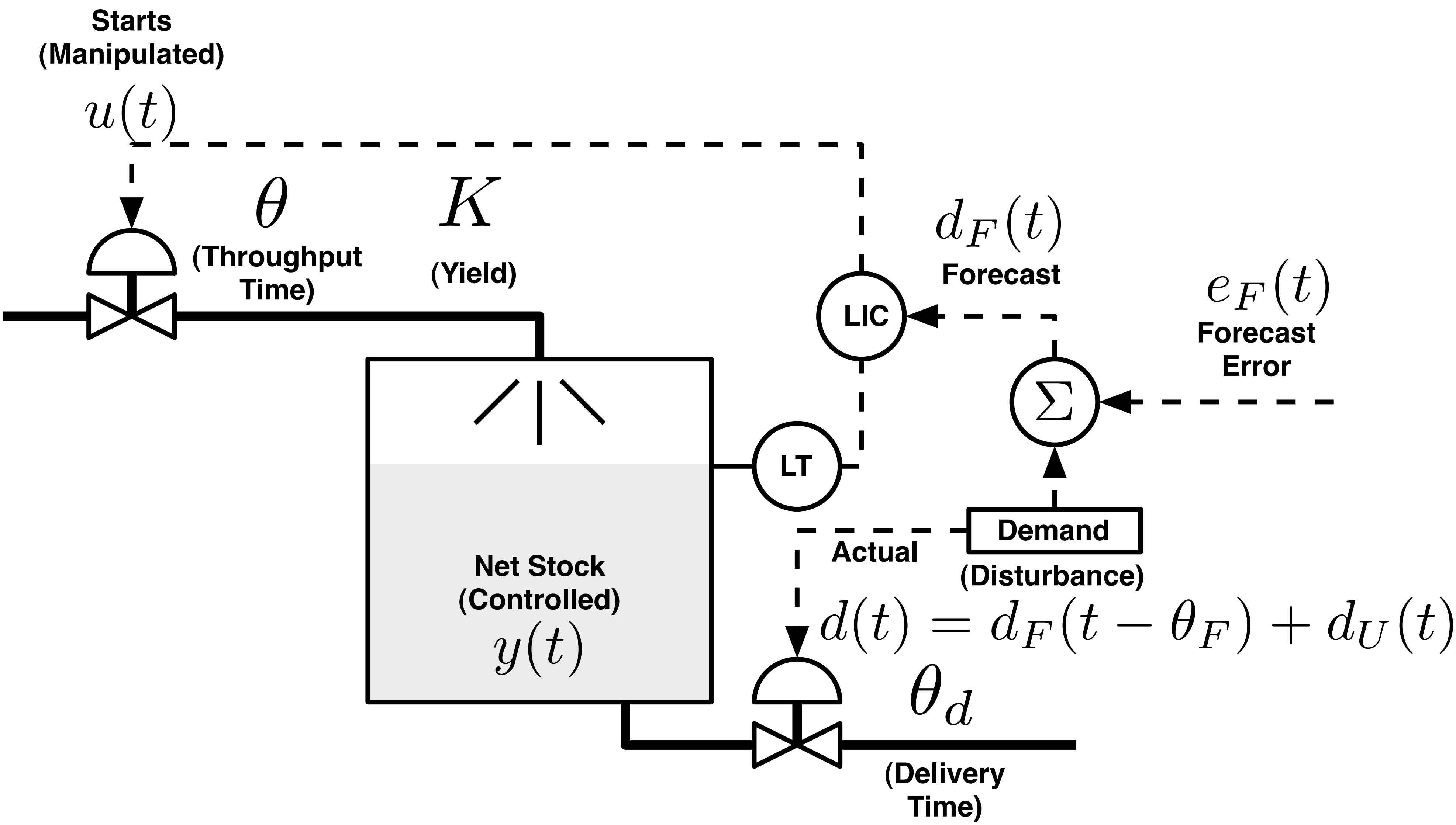
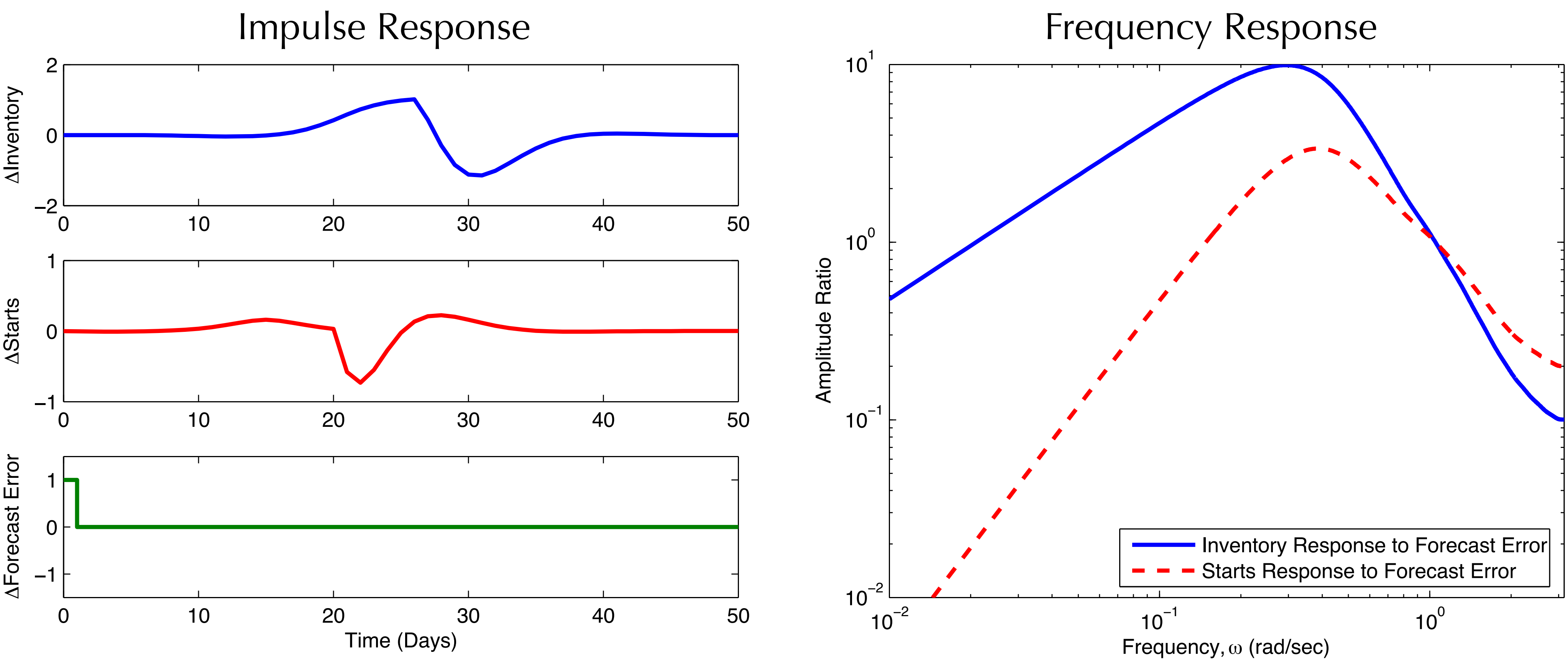
Production-Inventory System in the Presence of Forecast Error



Integrating System with Delays

System Response to Forecast Error

The closed-loop system response to a unit pulse in forecast error provides a basis for understanding modeling requirements for control-relevant demand models.



J.D. Schwartz and D.E. Rivera. "**A control-relevant approach to demand modeling for supply chain management,**" *Computers and Chemical Engineering*, 70:78-90, 2014.

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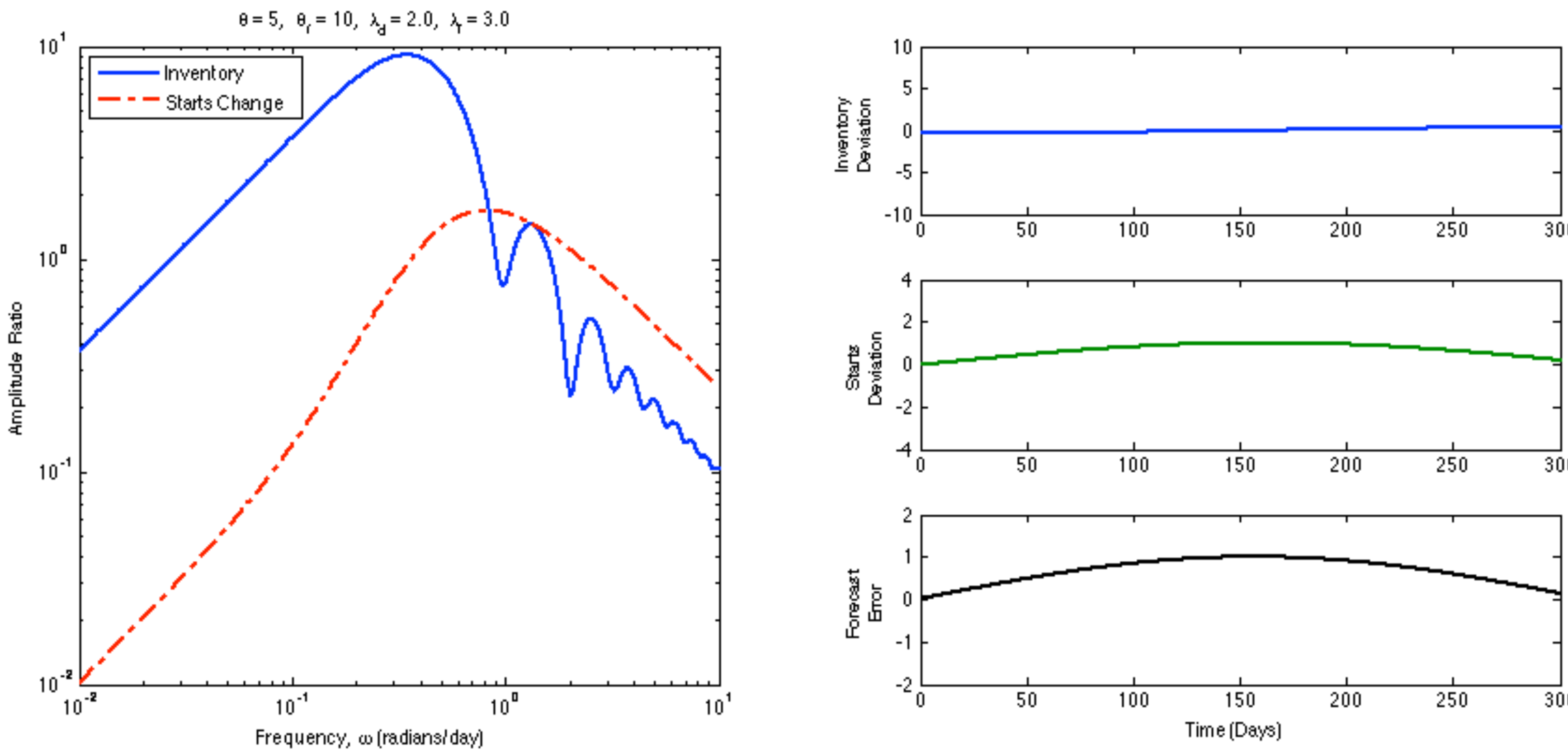
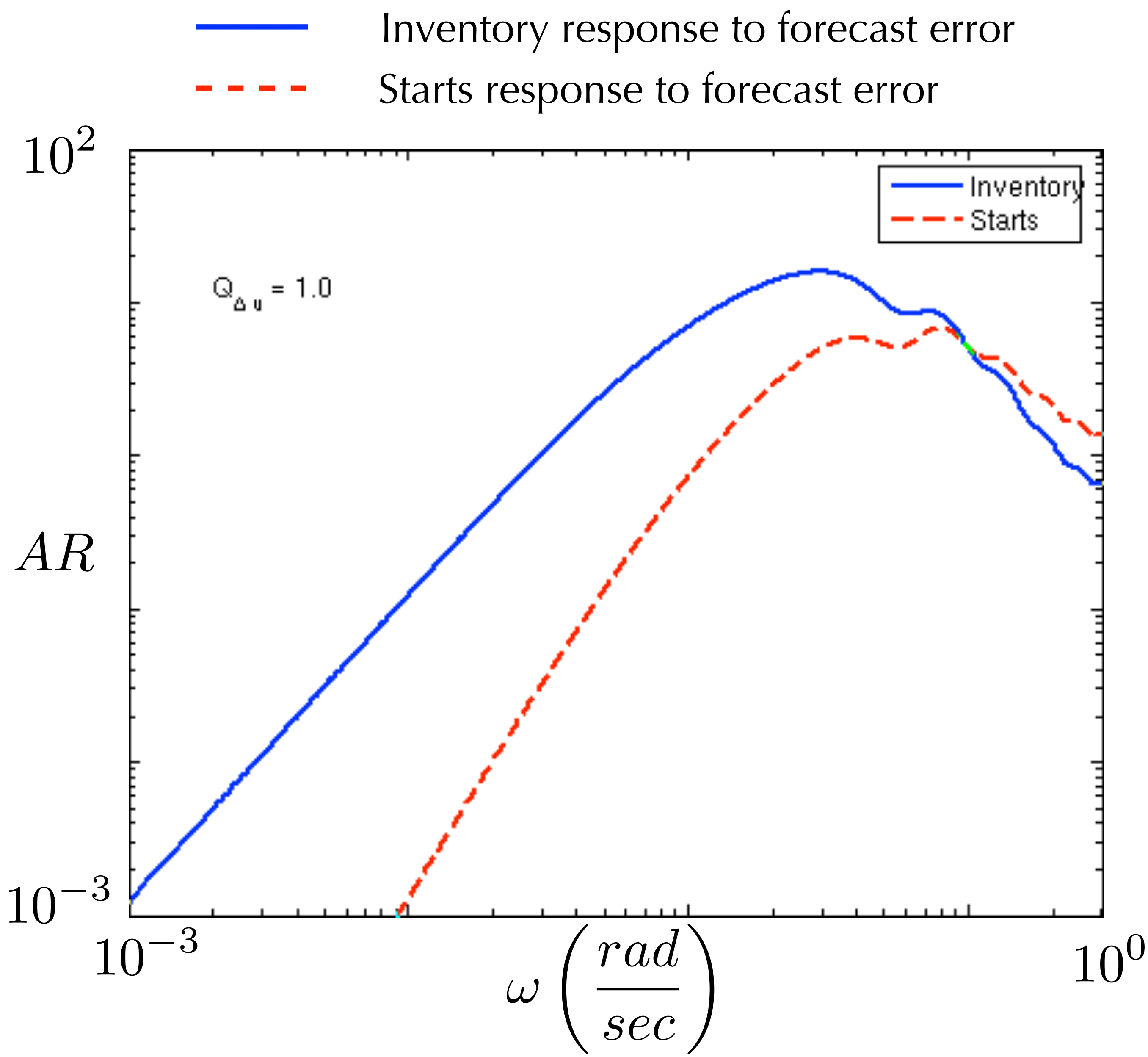
Understanding C-L Response to Forecast Error



The effect of forecast error on closed-loop performance is most significant in an intermediate frequency range.

Response to Forecast Error (MPC, changing move suppression)



Control-Relevant Estimation

True demand is defined by a demand transfer function $p_d(z)$ and a stochastic component $H(z)a(t)$.

$$d(t) = p_d(z)u_d(t) + H(z)a(t)$$

The estimated demand is defined by $\tilde{p}_d(z)$ and a noise model $\tilde{p}_e(z)$.

$$d(t) = \tilde{p}_d(z)u_d(t) + \tilde{p}_e e(t)$$

The control-relevant estimation step consists of minimizing the one-step-ahead prediction error, where $L(z)$ is the prefilter.

$$\min_{\tilde{p}_d, \tilde{p}_e} V = \min_{\tilde{p}_d, \tilde{p}_e} \frac{1}{N} \sum_{t=1}^{N} [L(z)e(t)]^2 = \min_{\tilde{p}_d, \tilde{p}_e} \frac{1}{N} \sum_{t=1}^{N} e_L^2(t)$$

Parseval's theorem allows for frequency domain analysis of the problem.

$$\lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} e_L^2(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{L(e^{j\omega})}{\tilde{p}_e(e^{j\omega})} \right|^2 \left(\left| p_d(e^{j\omega}) - \tilde{p}_d(e^{j\omega}) \right|^2 \Phi_{u_d}(\omega) + \left| H(e^{j\omega}) \right|^2 \Phi_a(\omega) \right) d\omega$$

Multi-Objective Formulation

It is desirable to minimize a weighted combination of inventory and factory starts variance.

$$\min_{\tilde{p}_d, \tilde{p}_e} \left[\sum_{t=0}^{\infty} (1-\gamma) e_c^2(t) + \lambda \sum_{t=0}^{\infty} \gamma \Delta u^2(t) \right]$$

The control-relevant prefilter then takes the following form.

$$\frac{|L(e^{j\omega})|^2}{|\tilde{p}_e(e^{j\omega})|^2} \Phi_{e_F}(\omega) = (1-\gamma)|L_{e_c}(e^{j\omega})|^2 \Phi_{e_F}(\omega) + \gamma\lambda |L_{\Delta u}(e^{j\omega})|^2 \Phi_{e_F}(\omega)$$

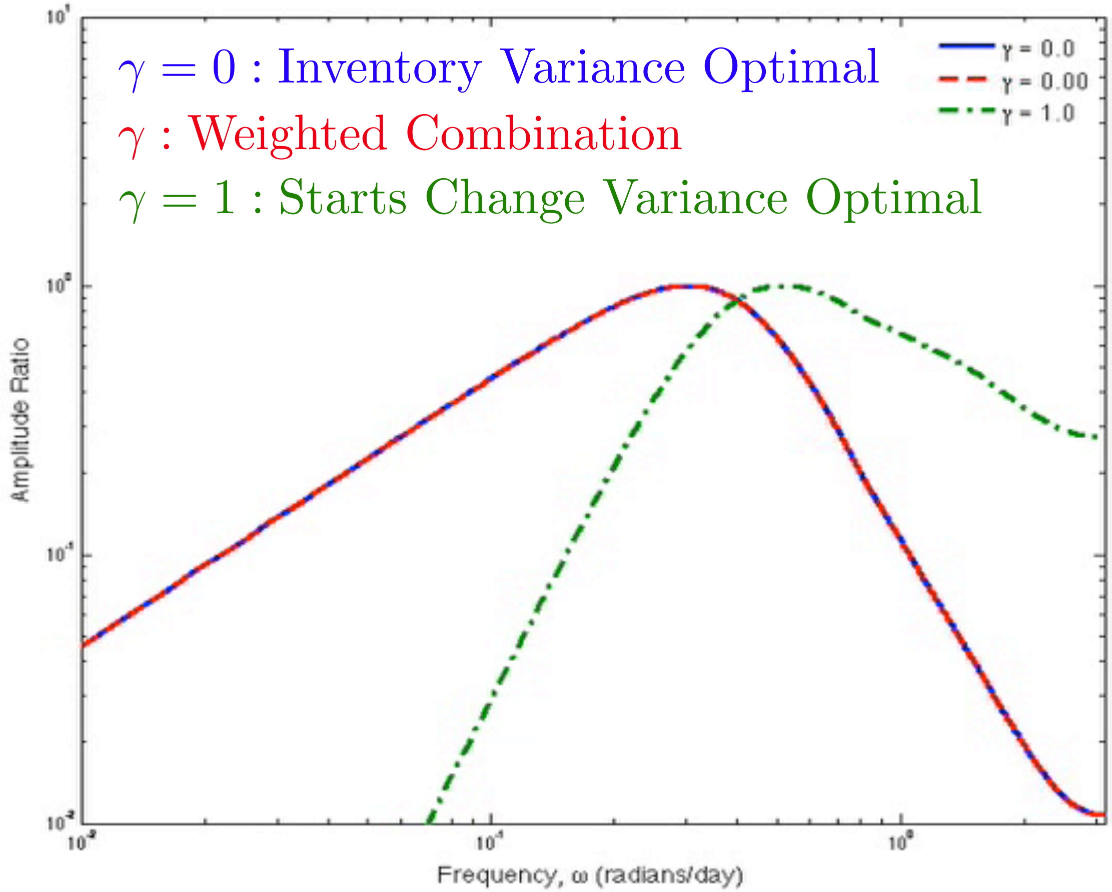
By assuming an output error model structure, $L(z)$ can be reduced to the following form.

$$|L(e^{j\omega})|^2 = (1-\gamma)|L_{e_c}(e^{j\omega})|^2 + \gamma\lambda |L_{\Delta u}(e^{j\omega})|^2$$

A curve fitting procedure is then used to obtain an Infinite Impulse Response filter that matches the amplitude ratio of the control-relevant prefilter.

Multi-Objective Formulation (Cont.)

$$|L(e^{j\omega})|^2 = (1-\gamma)|L_{e_c}(e^{j\omega})|^2 + \gamma\lambda|L_{\Delta u}(e^{j\omega})|^2$$



Final Observations

- Production-inventory systems are iconic dynamical systems that describe interesting problems in the process industries (and beyond).
- Combined feedback-feedforward strategies relying on demand forecast signals are necessary to adequately control these systems. Improved formulations of MPC can be developed in this regard.
- Demand modeling is a problem of significant importance in production-inventory systems; analysis of closed-loop decision policies show that these are most responsive to forecast error in an intermediate frequency bandwidth.
- Prefiltering can be used to apply the proper emphasis in control-relevant demand modeling.
- Multivariable extensions exist for both the control and demand modeling / demand forecasting segments of this presentation.

Primary References

- Schwartz, J.D., W. Wang, and D.E. Rivera, "Optimal tuning of process control policies for inventory management in supply chains," *Automatica*, 42, pgs. 1311- 1320, 2006.

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Additional references in http://csel.asu.edu/SCMpapers

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