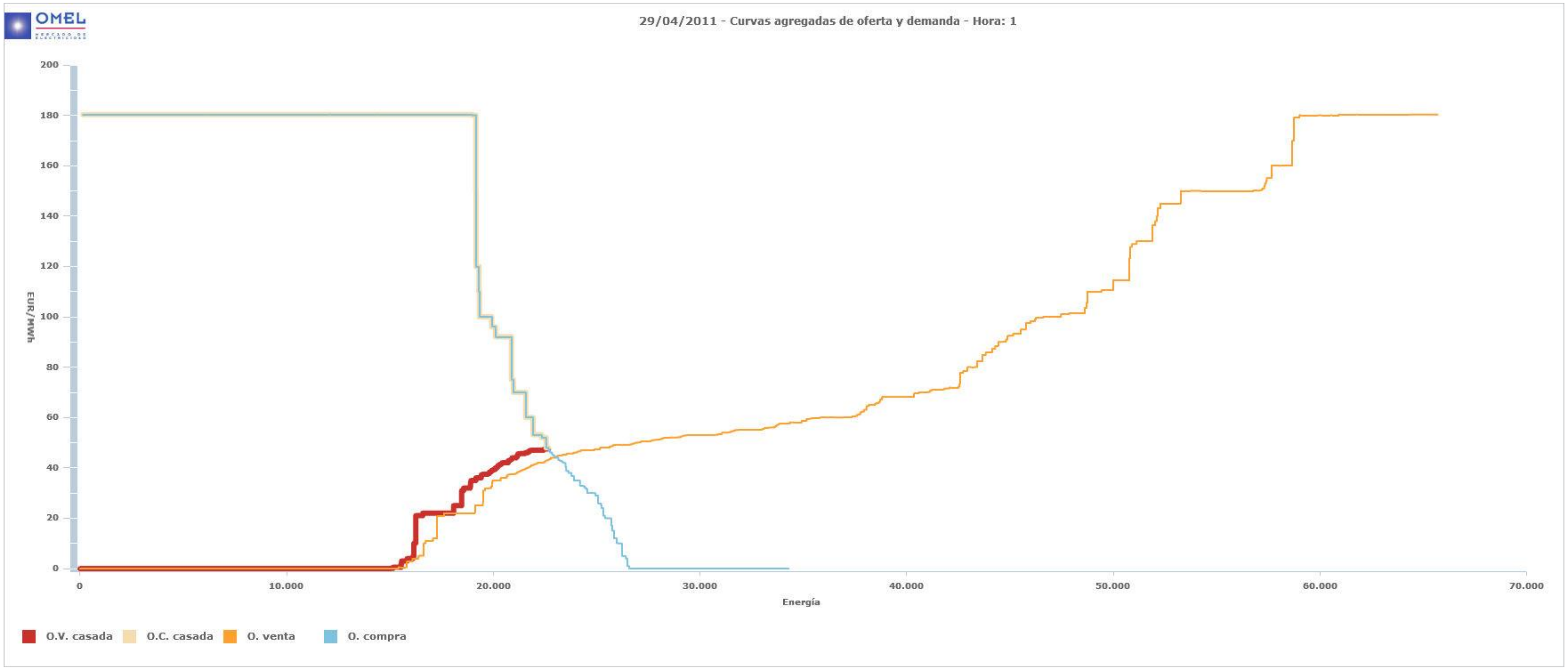
Pool strategy of an electricity producer with endogenous formation of clearing prices



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University of Castilla-La Mancha, Spain, 2011

Contents

- Background and Aim
- Approach
- Model Features
- Model Formulation
  - Deterministic
  - Stochastic
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Background and Aim

Strategic power producer

- Comparatively large number of generating units
- Units distributed throughout the power network

Background and Aim

Pool-based electricity market

- Cleared once a day, one-day ahead and on a hourly basis
- DC representation of the network including first and second Kirchhoff laws
- Hourly Locational Marginal Prices (LMPs)

Background and Aim

Strategic power producer

↓ Best offering strategy to maximize profit

Pool-based electricity market

Background and Aim

- Considering the market: MPEC formulation
- Considering the real-world: Stochastic formulation

- Stochastic MPEC!

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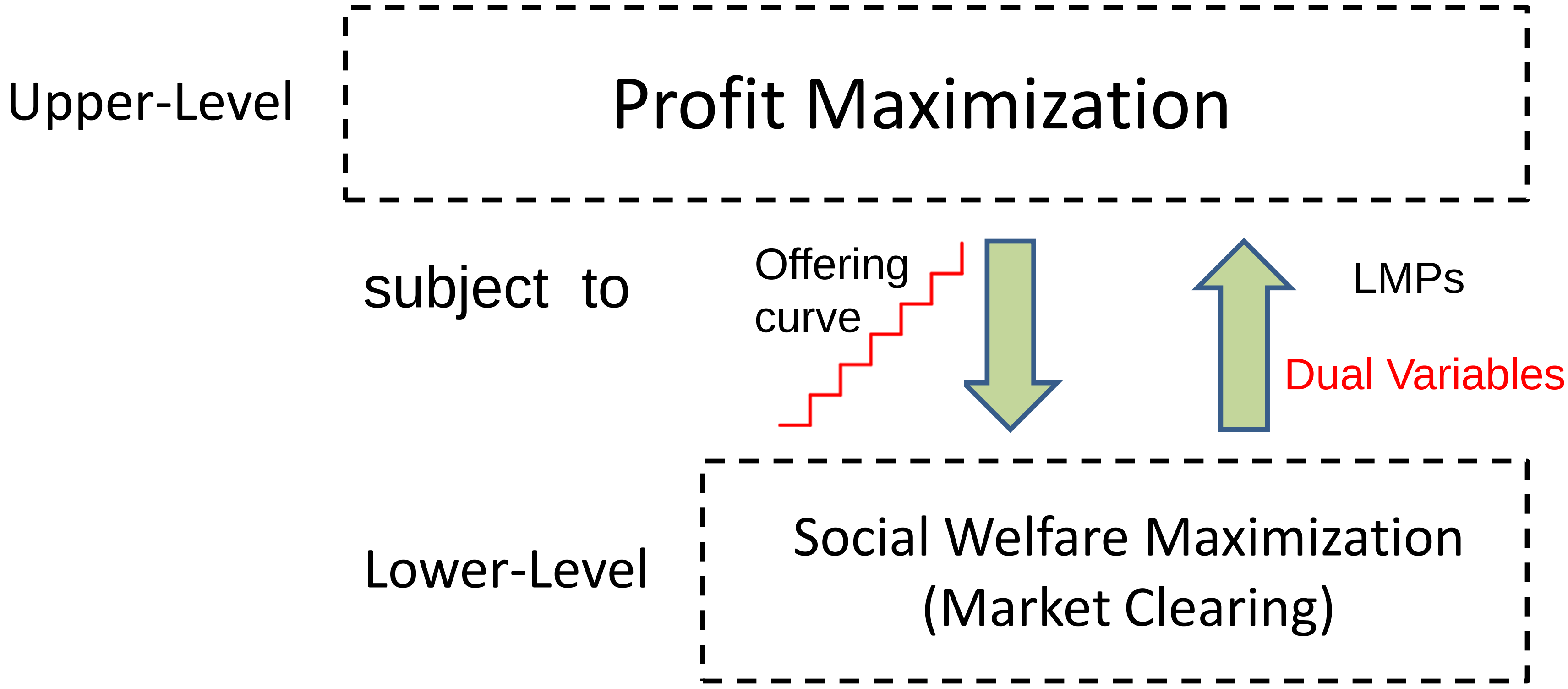
Approach

Bilevel model:



Approach

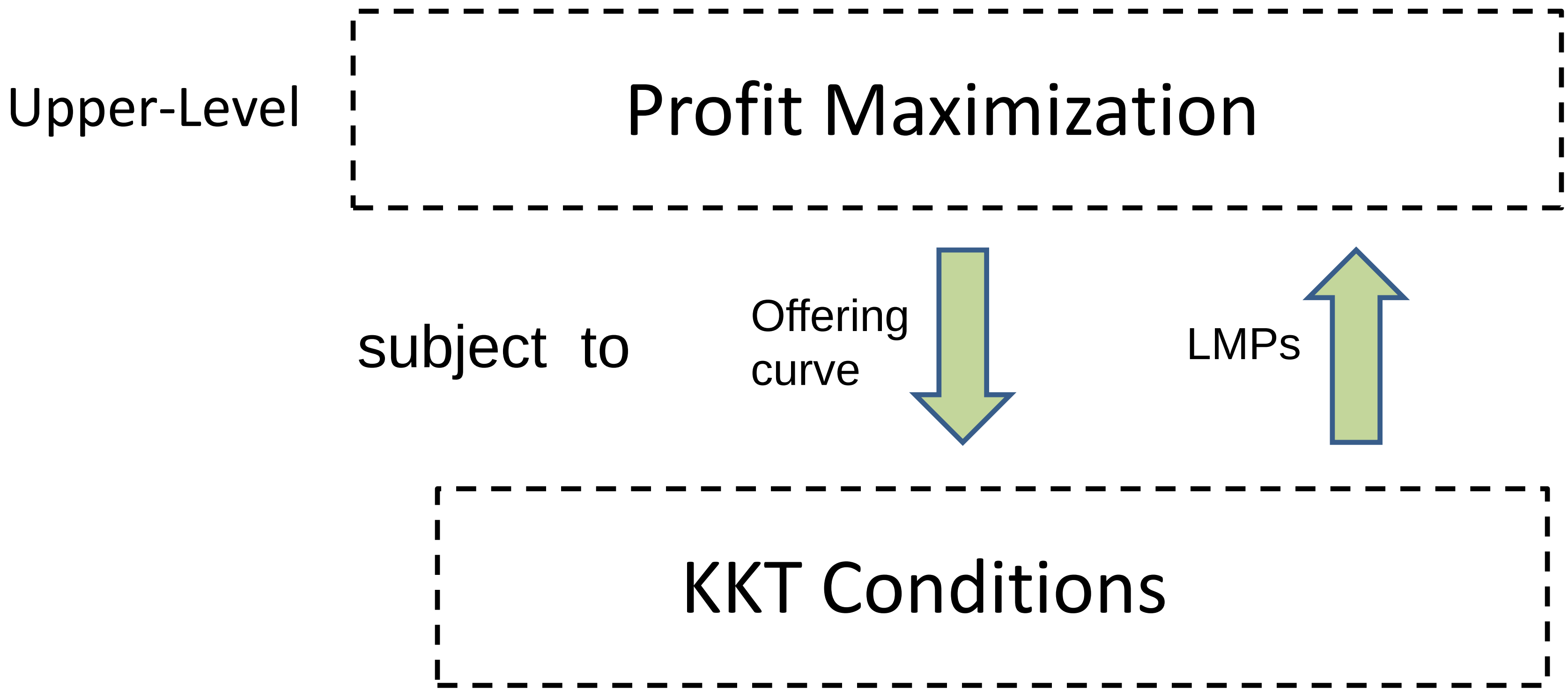
- Bilevel model: Optimization problem constrained by other optimization problem (OPcOP)!

OPcOP

$$
\begin{aligned}
&\underset{x,y,\lambda,\mu}{\text{minimize}} \quad f^{\mathrm{U}}(x, y, \lambda, \mu) \\
&\text{subject to} \\
&\quad h^{\mathrm{U}}(x, y, \lambda, \mu) = 0 \\
&\quad g^{\mathrm{U}}(x, y, \lambda, \mu) \leq 0, \\
&\quad \left\{ \begin{array}{ll}
\underset{y}{\text{minimize}} \quad c(x)^{\mathrm{T}} y & \\
\text{subject to} & \\
\quad D(x)y = e(x) & : \lambda \\
\quad A(x)y \leq b(x) & : \mu,
\end{array} \right.
\end{aligned}
$$

Approach

MPEC:



MPEC

$$\begin{aligned}
&\underset{x,y,\lambda,\mu}{\text{minimize}} \quad f^{\mathrm{U}}(x,y,\lambda,\mu) \\
&\text{subject to} \\
&\quad h^{\mathrm{U}}(x,y,\lambda,\mu) = 0 \\
&\quad g^{\mathrm{U}}(x,y,\lambda,\mu) \le 0, \\
&\quad c(x) + A(x)^{\mathrm{T}}\mu - D(x)^{\mathrm{T}}\lambda = 0, \\
&\quad D(x)y = e(x), \\
&\quad 0 \le (b - A(x)y) \perp \mu \ge 0, \\
&\quad \lambda \quad : \ \text{free.}
\end{aligned}$$

MPEC

$$
\begin{aligned}
\underset{x,y,\lambda,\mu}{\text{minimize}} \quad & f^{\mathrm{U}}(x, y, \lambda, \mu) \\
\text{subject to} \quad & \\
& h^{\mathrm{U}}(x, y, \lambda, \mu) = 0, \\
& g^{\mathrm{U}}(x, y, \lambda, \mu) \leq 0, \\
& c(x)^{\mathrm{T}} y = -b(x)^{\mathrm{T}} \mu + e(x)^{\mathrm{T}} \lambda, \\
& D(x) y = e(x), \\
& A(x) y \leq b(x), \\
& -A(x)^{\mathrm{T}} \mu + D(x)^{\mathrm{T}} \lambda = c(x), \\
& \mu \geq 0, \\
& \lambda \quad : \text{ free.}
\end{aligned}
$$

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Features

1) Strategic offering for a producer in a pool with endogenous formation of LMPs.
2) Uncertainty of demand bids and rival production offers.
3) MPEC approach under multi-period, network-constrained pool clearing.
4) MPEC transformed into an equivalent MILP.

Contents

Deterministic Model

Upper-Level → Profit Maximization:

Minimize

Costs - Revenues

subject to:

Ramping Limits

Price = Balance dual variable

Deterministic Model

Upper-Level → Profit Maximization:

Dual variable

$$\underset{\alpha^S_{tib}, P^S_{tib}, \forall t, \forall i, \forall b}{\text{Minimize}} \quad \sum_{tib} \lambda^S_{tib} P^S_{tib} - \sum_{t(i \in \Psi_n)b} \beta_{tn} P^S_{tib}$$

subject to:

$$\sum_b P^S_{(t+1)ib} - \sum_b P^S_{tib} \leq R_i^{\text{UP}} \qquad \forall t < T, \forall i$$

$$\sum_b P^S_{tib} - \sum_b P^S_{(t+1)ib} \leq R_i^{\text{LO}} \qquad \forall t < T, \forall i$$

$$\beta_{tn} = \lambda_{tn} \qquad \forall t, \forall n$$

Deterministic Model

Lower-Level → Market Clearing

$P^S_{tib} \in \arg$

Maximize Social Welfare

subject to:

Power Balance

Deterministic Model

Lower-Level → Market Clearing

$$P^S_{tib} \in \arg \left\{ \underset{P^S_{tib}, P^O_{tjb}, P^D_{tdk}}{\text{Minimize}} \quad \sum_{tib} \alpha^S_{tib} P^S_{tib} + \sum_{tjb} \lambda^O_{tjb} P^O_{tjb} - \sum_{tdk} \lambda^D_{tdk} P^D_{tdk} \right.$$

subject to:

$$\left. \sum_{(i \in \Psi_n) b} P^S_{tib} + \sum_{(j \in \Psi_n) b} P^O_{tjb} - \sum_{(d \in \Psi_n) k} P^D_{tdk} = = \sum_{m \in \Theta_n} B_{nm} (\delta_{tn} - \delta_{tm}) \quad : \lambda_{tn} \quad \forall t, \forall n \right\}$$

Price

Deterministic Model

Lower-Level → Market Clearing

subject to:

Production / Demand Power Limits

Transmission Capacity Limits

Angle Limits

Deterministic Model

Lower-Level → Market Clearing

subject to:

$$0 \leq P^S_{tib} \leq P^{S^{\max}}_{tib} \qquad : \mu^{S^{\min}}_{tib}, \mu^{S^{\max}}_{tib} \qquad \forall t, \forall i, \forall b$$

$$0 \leq P^O_{tjb} \leq P^{O^{\max}}_{tjb} \qquad : \mu^{O^{\min}}_{tjb}, \mu^{O^{\max}}_{tjb} \qquad \forall t, \forall j, \forall b$$

$$0 \leq P^D_{tdk} \leq P^{D^{\max}}_{tdk} \qquad : \mu^{D^{\min}}_{tdk}, \mu^{D^{\max}}_{tdk} \qquad \forall t, \forall d, \forall k$$

$$-C^{max}_{nm} \leq B_{nm}(\delta_{tn} - \delta_{tm}) \leq C^{\max}_{nm} \qquad : \nu^{\min}_{tnm}, \nu^{\max}_{tnm}$$
$$\forall t, \forall n, \forall m \in \Theta_n$$

$$-\pi \leq \delta_{tn} \leq \pi \qquad : \xi^{\min}_{tn}, \xi^{\max}_{tn} \qquad \forall t, \forall n$$

$$\delta_{tn} = 0 \qquad : \xi^1_t \qquad \forall t, n = 1$$

Deterministic Model

Lower-Level → Market Clearing → KKT conditions

$$\alpha_{tib}^{S} - \lambda_{tn} + \mu_{tib}^{S^{\max}} - \mu_{tib}^{S^{\min}} = 0 \qquad \forall t, \forall i \in \Psi_n, \forall b$$

$$\lambda_{tjb}^{O} - \lambda_{tn} + \mu_{tjb}^{O^{\max}} - \mu_{tjb}^{O^{\min}} = 0 \qquad \forall t, \forall j \in \Psi_n, \forall b$$

$$-\lambda_{tdk}^{D} + \lambda_{tn} + \mu_{tdk}^{D^{\max}} - \mu_{tdk}^{D^{\min}} = 0 \quad \forall t, \forall d \in \Psi_n, \forall k$$

$$\sum_{m \in \Theta_n} B_{nm}(\lambda_{tn} - \lambda_{tm}) + \sum_{m \in \Theta_n} B_{nm}(\nu_{tnm}^{\max} - \nu_{tmn}^{\max})$$
$$+ \sum_{m \in \Theta_n} B_{nm}(\nu_{tmn}^{\min} - \nu_{tnm}^{\min}) + \xi_{tn}^{\max} - \xi_{tn}^{\min} + (\xi_t^{1})_{n=1} = 0 \quad \forall t, \forall n$$

$$\sum_{ib} P_{t(i \in \Psi_n),b}^{S} + \sum_{jb} P_{t(j \in \Psi_n)b}^{O} - \sum_{dk} P_{t(d \in \Psi_n)k}^{D} =$$
$$= \sum_{m \in \Theta_n} B_{nm}(\delta_{tn} - \delta_{tm}) \qquad \forall t, \forall n$$

$$\delta_{tn} = 0 \qquad \forall t, n = 1$$

Deterministic Model

Lower-Level → Market Clearing → KKT conditions

$$0 \leq P^{S}_{tib} \perp \mu^{S^{\min}}_{tib} \geq 0 \qquad \forall t, \forall i, \forall b$$

$$0 \leq P^{O}_{tjb} \perp \mu^{O^{\min}}_{tjb} \geq 0 \qquad \forall t, \forall j, \forall b$$

$$0 \leq P^{D}_{tdk} \perp \mu^{D^{\min}}_{tdk} \geq 0 \qquad \forall t, \forall d, \forall k$$

$$0 \leq P^{S^{\max}}_{tib} - P^{S}_{tib} \perp \mu^{S^{\max}}_{tib} \geq 0 \qquad \forall t, \forall i, \forall b$$

$$0 \leq P^{O^{\max}}_{tjb} - P^{O}_{tjb} \perp \mu^{O^{\max}}_{tjb} \geq 0 \qquad \forall t, \forall j, \forall b$$

$$0 \leq P^{D^{\max}}_{tdk} - P^{D}_{tdk} \perp \mu^{D^{\max}}_{tdk} \geq 0 \qquad \forall t, \forall d, \forall k$$

$$0 \leq C^{\max}_{nm} + B_{nm}(\delta_{tn} - \delta_{tm}) \perp \nu^{\min}_{tnm} \geq 0$$
$$\forall t, \forall n, \forall m \in \Theta_n$$

$$0 \leq C^{\max}_{nm} - B_{nm}(\delta_{tn} - \delta_{tm}) \perp \nu^{\max}_{tnm} \geq 0$$
$$\forall t, \forall n, \forall m \in \Theta_n$$

$$0 \leq \pi - \delta_{tn} \perp \xi^{\max}_{tn} \geq 0 \qquad \forall t, \forall n$$

$$0 \leq \pi + \delta_{tn} \perp \xi^{\min}_{tn} \geq 0 \qquad \forall t, \forall n$$

Deterministic Model

MPEC model

$$\text{Minimize} \quad \sum_{tib} \lambda^S_{tib} P^S_{tib} - \sum_{t(i \in \Psi_n)nb} \lambda_{tn} P^S_{tib}$$

subject to:

$$\sum_b P^S_{(t+1)ib} - \sum_b P^S_{tib} \leq R_i^{UP} \qquad \forall t, \forall i$$

$$\sum_b P^S_{tib} - \sum_b P^S_{(t+1)ib} \leq R_i^{LO} \qquad \forall t, \forall i$$

KKT Lower-Level

Deterministic Model

Linearizations

The MPEC includes the following non-linearities:

1) The complementarity conditions ($0 \leq a \perp b \geq 0$).

2) The term $\lambda_{tn} P_{tib}^{S}$ in the objective function.

Deterministic Model

Linearizations → Complementarity Conditions

Fortuny-Amat transformation

$$0 \le a \perp b \ge 0$$

$$\Downarrow$$

$$a \ge 0$$

$$b \ge 0$$

$$a \le uM$$

$$b \le (1-u)M$$

$$u \in \{0,1\}$$

M Large enough constant (but not too large)

Deterministic Model

Linearizations → Term: $\lambda_{tn} P^S_{tib}$

Based on the strong duality theorem and some of the KKT equalities

$$X = \sum_{t(i \in \Psi_n)b} \lambda_{tn} P^S_{tib} = -\sum_{tjb} \lambda^O_{tjb} P^O_{tjb} + \sum_{tdh} \lambda^D_{tdk} P^D_{tdk}$$
$$-\sum_{tjb} \mu^{O^{\max}}_{tjb} P^{O^{\max}}_{tjb} - \sum_{tdk} \mu^{D^{\max}}_{tdk} P^{D^{\max}}_{tdk} - \sum_{tn(m \in \Theta_n)} \nu^{\min}_{tnm} C^{\max}_{nm}$$
$$- \sum_{tn(m \in \Theta_n)} \nu^{\max}_{tnm} C^{\max}_{nm} - \sum_{tn} \xi^{\max}_{tn} \pi - \sum_{tn} \xi^{\min}_{tn} \pi$$

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Stochastic Model

Uncertainty incorporated using a set of scenarios modeling different realizations of:

- Consumers’ bids
- Rival producers’ offers

Stochastic Model

Deterministic model for each scenario

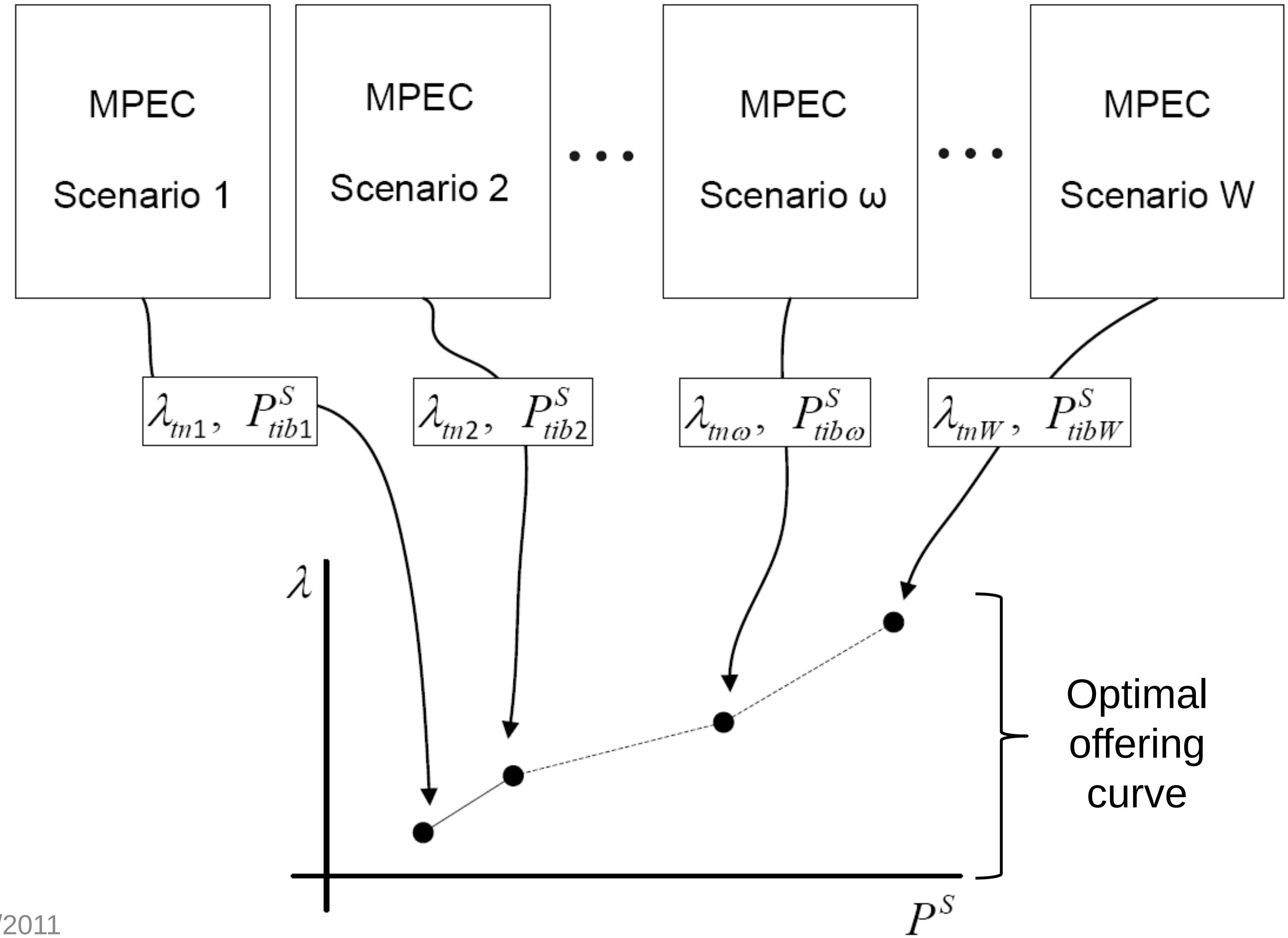
↓

Pairs of production quantities ($P^S_{tib\omega}$) and market prices ($\lambda_{tn\omega}$).

↓

Building of the optimal offering curve

Stochastic Model



Stochastic Model

To ensure that the final offering curves are increasing in price some additional constraints are needed:

$$\lambda_{tnw} - \lambda_{tnw'} \leq x_{tiww'} M^x \quad \forall t, \forall i \in \Psi_n, \forall w, \forall w' > w$$

$$\lambda_{tnw} - \lambda_{tnw'} \geq (x_{tiww'} - 1) M^x \quad \forall t, \forall i \in \Psi_n, \forall w, \forall w' > w$$

$$\sum_b P^S_{tibw} - \sum_b P^S_{tibw'} \leq y_{tiww'} M^y \quad \forall t, \forall i, \forall w, \forall w' > w$$

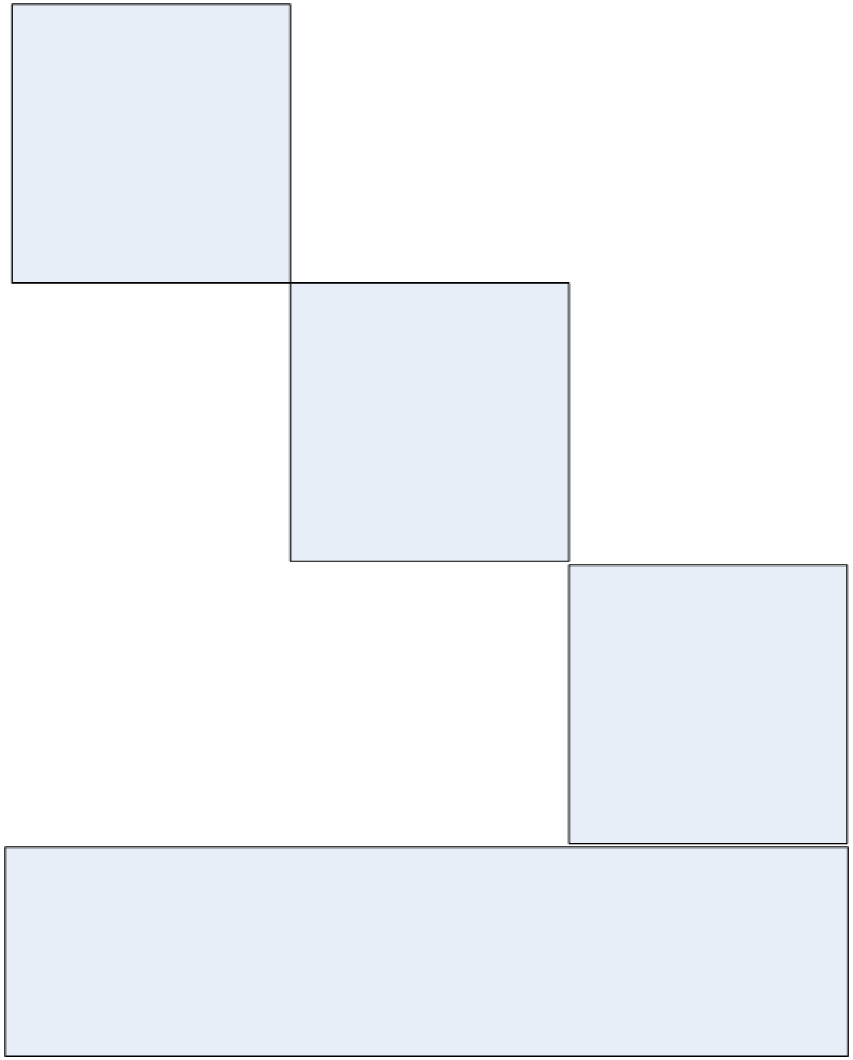
$$\sum_b P^S_{tibw} - \sum_b P^S_{tibw'} \geq (y_{tiww'} - 1) M^y \quad \forall t, \forall i, \forall w, \forall w' > w$$

$$x_{tiww'} + y_{tiww'} = 2 z_{tiww'} \quad \forall t, \forall i, \forall w, \forall w' > w$$

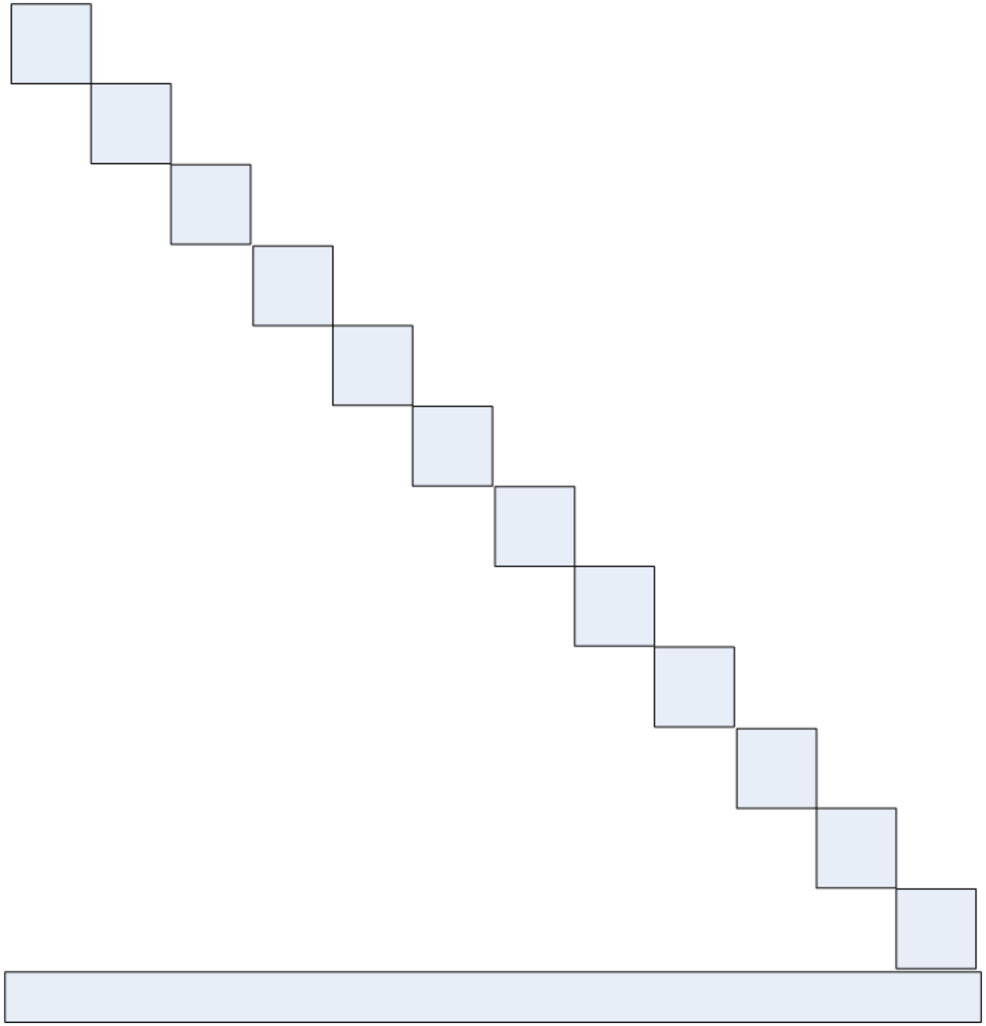
$$x_{tiww'}, y_{tiww'}, z_{tiww'} \in \{0, 1\}$$

These constraints link the individual problems increasing the computational complexity of the model.

Stochastic Model
Math Structure



Stochastic Model
Math Structure



Stochastic Model
Math Structure

1. Direct solution: CPLEX, XPRESS

2. Decomposition procedures (Lagrangian Relaxation)

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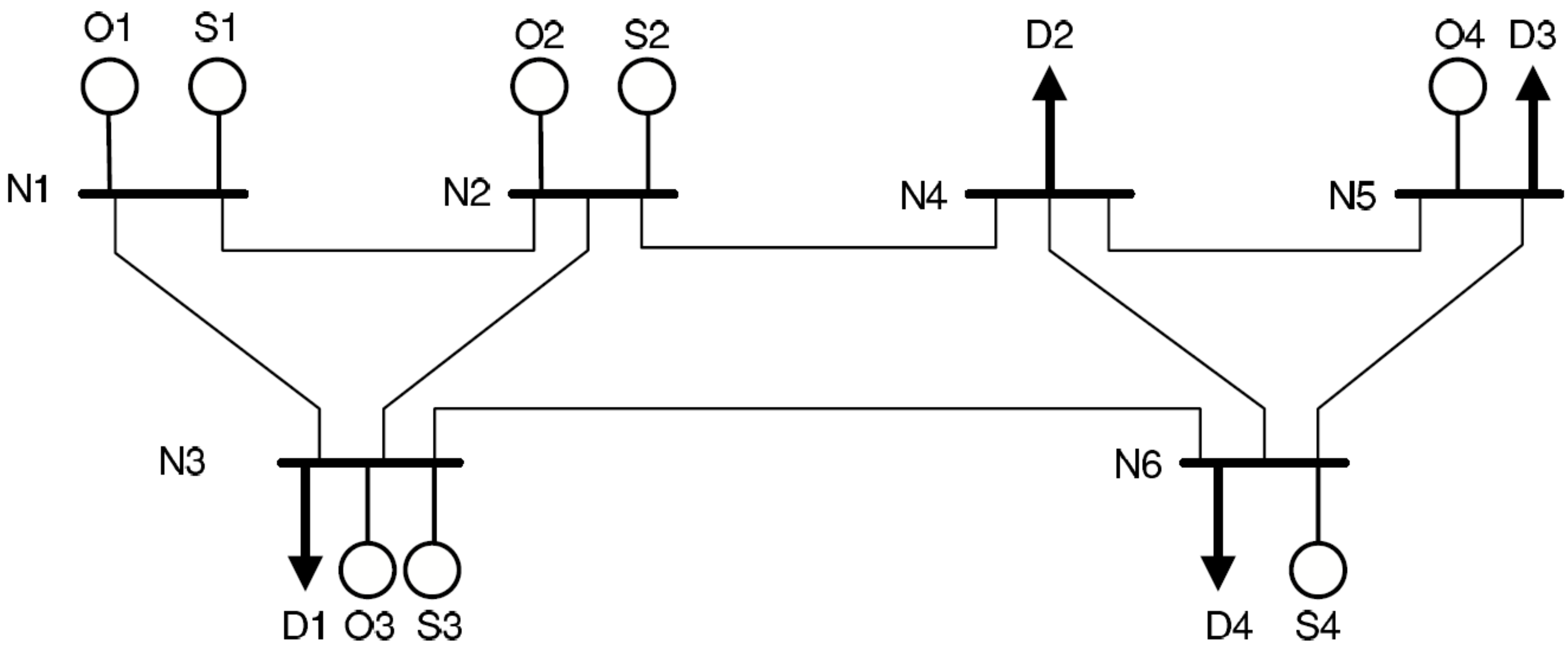
Examples

Six-bus test system→ electricity network



Examples

Six-bus test system→ demand curve

DEMAND BLOCKS [GWH] FOR EACH PERIOD OF TIME

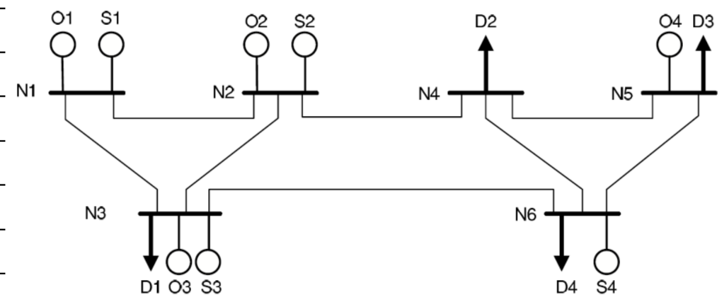
[€/MWh]	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
25.000												0.9	0.9					0.9	0.9	0.9	0.9			
24.968											0.9	0.025	0.025	0.9				0.025	0.025	0.025	0.025	0.9		
22.628											0.025	0.025	0.025	0.025				0.025	0.025	0.025	0.025	0.025		
20.876											0.025	0.025	0.025	0.025			0.9	0.025	0.025	0.025	0.025	0.025		
20.606											0.025	0.025	0.025	0.025			0.025	0.025	0.025	0.025	0.025	0.025		
20.378										0.9	0.025			0.025	0.9	0.9	0.025					0.025		
19.922										0.025					0.025	0.025	0.025							
19.532										0.025					0.025	0.025	0.025						0.9	
19.232									0.9	0.025					0.025	0.025							0.025	
18.932									0.025	0.025					0.025	0.025							0.025	
18.806									0.025														0.025	
18.344									0.025														0.025	
18.152									0.025															
17.940								0.9																0.9
17.612								0.025																0.025
17.430	0.9							0.025																0.025
17.250	0.025	0.9					0.9	0.025																0.025
17.216	0.025	0.025	0.9	0.9			0.025	0.025																0.025
16.886	0.025	0.025	0.025	0.025	0.9	0.9	0.025																	
16.790	0.025	0.025	0.025	0.025	0.025	0.025	0.025																	
16.380		0.025	0.025	0.025	0.025	0.025	0.025																	
16.320			0.025	0.025	0.025	0.025																		
16.130					0.025	0.025																		

Examples

Six-bus test system→ generating units

TYPE AND DATA FOR THE GENERATING UNITS

Unit Type	oil	oil	hydro	coal	oil	coal	oil	coal	nuclear
P[MW]	12	20	50	76	100	155	197	350	400
$P_1^{\max}$[MWh]	2.4	15.8	15	15.2	25	54.25	68.95	140	100
$P_2^{\max}$[MWh]	3.4	0.2	15	22.8	25	38.75	49.25	97.5	100
$P_3^{\max}$[MWh]	3.6	3.8	10	22.8	20	31	39.4	52.5	120
$P_4^{\max}$[MWh]	2.4	0.2	10	15.2	20	31	39.4	70	80
$\lambda_1^{S/O}$[€/MWh]	23.41	11.09	0	11.46	18.60	9.92	10.08	19.20	5.31
$\lambda_2^{S/O}$[€/MWh]	23.78	11.42	0	11.96	20.03	10.25	10.66	20.32	5.38
$\lambda_3^{S/O}$[€/MWh]	26.84	16.06	0	13.89	21.67	10.68	11.09	21.22	5.53
$\lambda_4^{S/O}$[€/MWh]	30.40	16.24	0	15.97	22.72	11.26	11.72	22.13	5.66
R^{UP} [MW]	30	90	-	60	210	90	90	120	600
R^{LO} [MW]	30	90	-	60	210	90	90	120	600



LOCATION AND TYPE OF UNITS

Strategic units			Other units		
i	Type	Bus	j	Type	Bus
1	155	1	1	350	1
2	100	2	2	197	2
3	155	3	3	197	3
4	197	6	4	155	5

LOCATION AND DISTRIBUTION OF THE DEMAND

d	Bus	Factor (%)
1	3	19
2	4	27
3	5	27
4	6	27

Examples

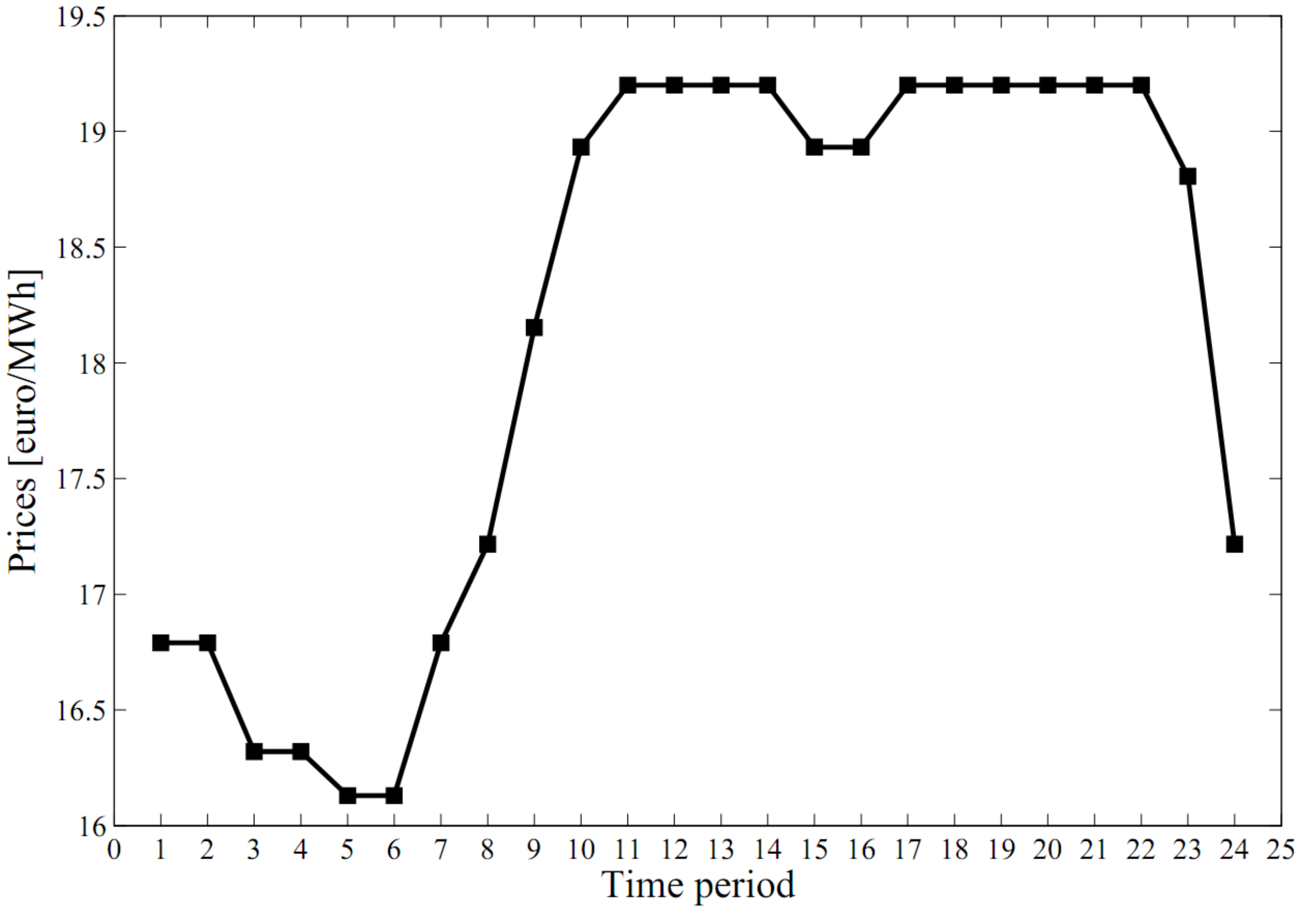
Six-bus test system→ uncongested network results

Strategic Offer					
	S1	S2	S3	S4	Total
Production [MWh]	3501.8	0	3464.8	3782.4	10749
Profit [€]	27202	0	27038	28861	83101
Marginal-Cost Offer					
Production [MWh]	3720	0	3720	3805.2	11245.2
Profit [€]	4826.7	0	4826.7	4562.5	14216

The maximum power flow through lines 2-4, 3-6 and 4-6 are 269.62, 229.44 and 39.6933 MW respectively

Examples

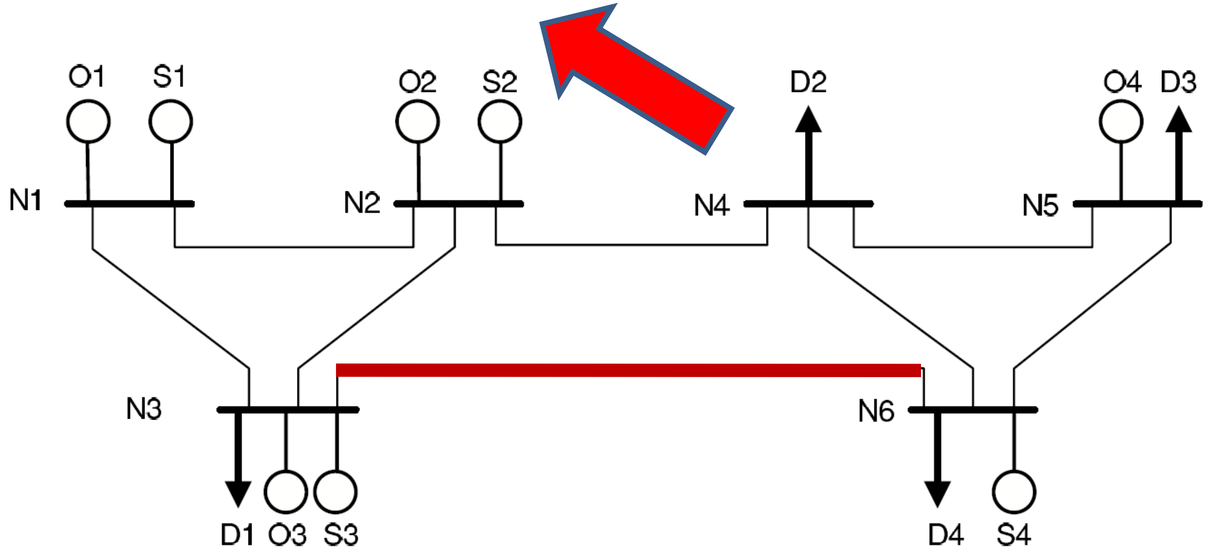
Six-bus test system→ uncongested network results



Examples

Six-bus test system→ congested network results

Capacity of line 3-6 limited to 230 MW:

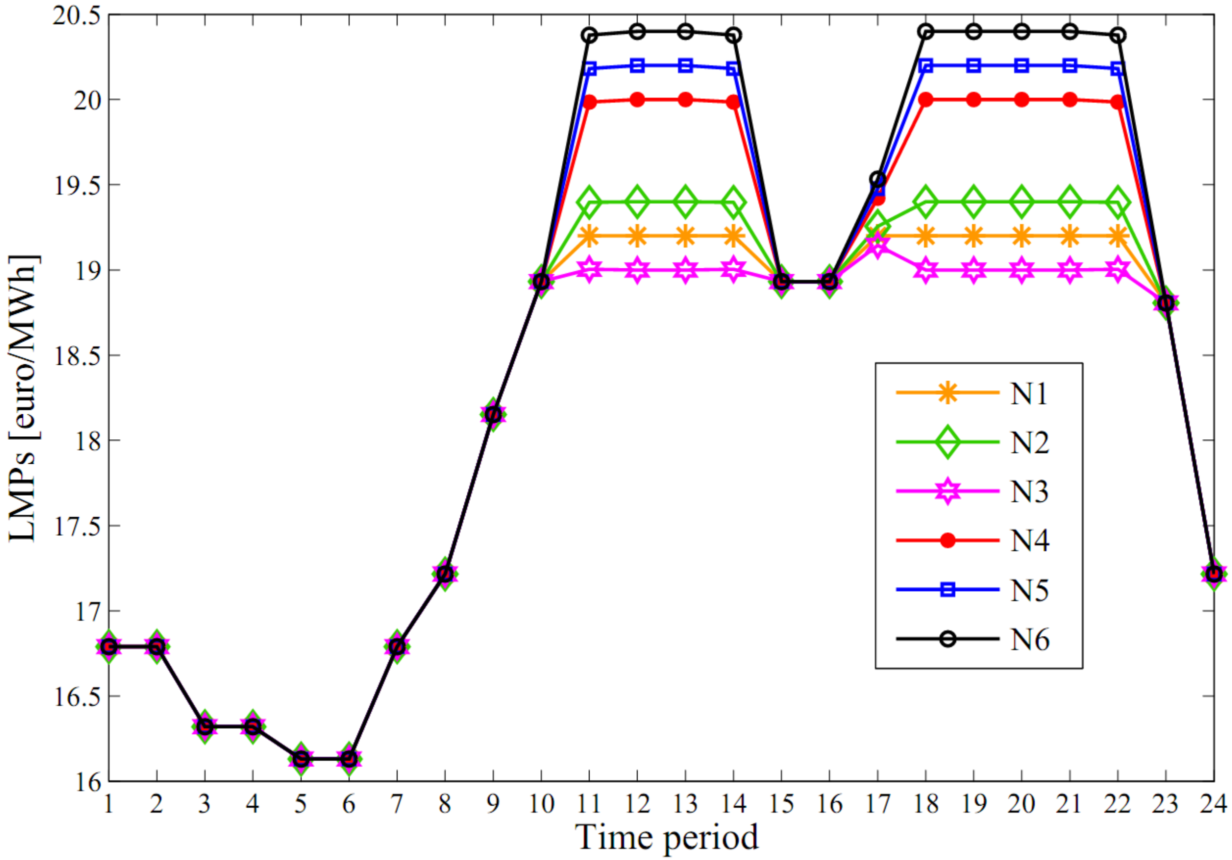


	S1	S2	S3	S4	Total
Production [MWh]	3477.9	0	3498	3773.1	10749
Profit [€]	2691	0	27068	30519	84574

Examples

Six-bus test system→ congested network results

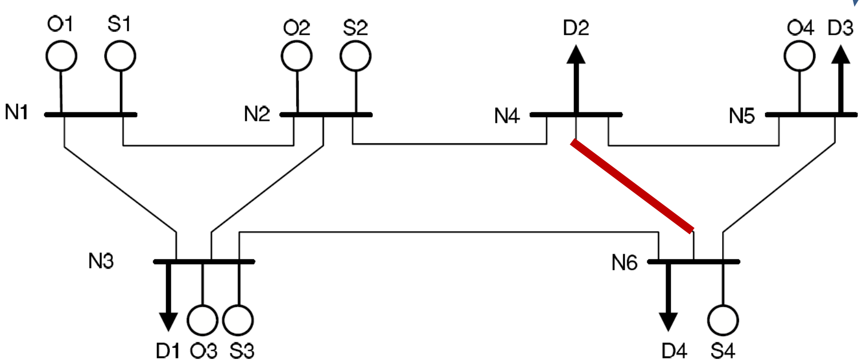
Capacity of line 3-6 limited to 230 MW:

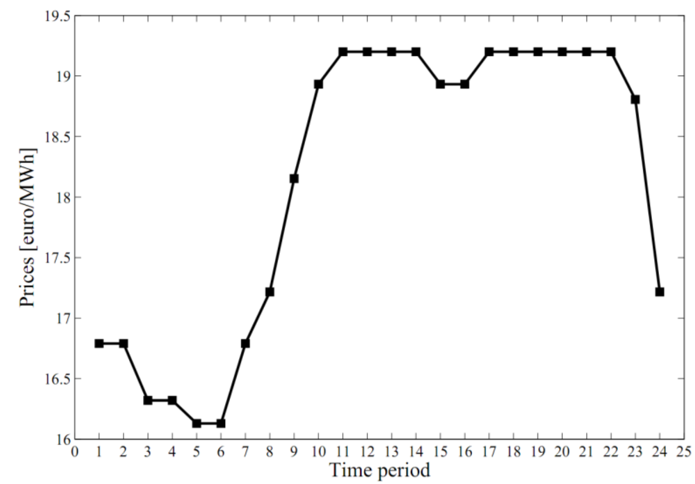


Examples

Six-bus test system→ congested network results

Capacity of line 4-6 limited to 39 MW:





	S1	S2	S3	S4	Total
Production [MWh]	3610.3	0	3356.3	3782.4	10749
Profit [€]	28027	0	26214	28861	83101

Examples

Six-bus test system→ stochastic model

- Uncongested network case
- 8 equally probable scenarios
- They differ on the rival producer offers ($\lambda^{O}_{tjb\omega}$) and on the consumer bids ($\lambda^{D}_{tdk\omega}$)
- Selected to obtain a wide range of prices

Examples

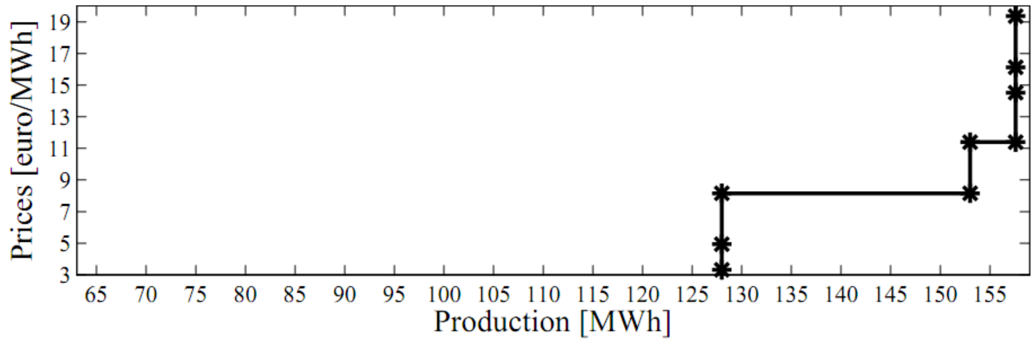
Six-bus test system→ stochastic model results

Strategic Offer					
	S1	S2	S3	S4	Total
E. Production [MWh]	3088.3	0	3008.6	3326.7	9423.6
E. Profit [€]	16354	0	15657	16615	48626
Marginal-Cost Offer					
E. Production [MWh]	2331.4	0	2437.6	2715.2	7484.2
E. Profit [€]	6430.4	0	6430.4	7281.1	20141.9

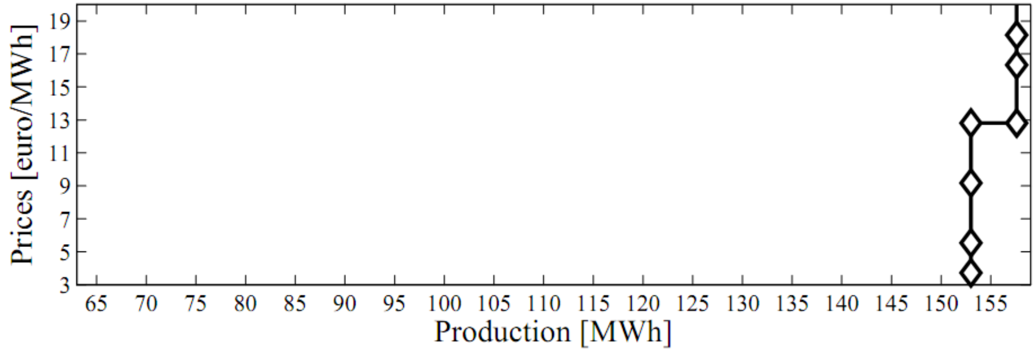
Examples

Six-bus test system→ stochastic model results

Offering curves for strategic generator 1



(a) t=5

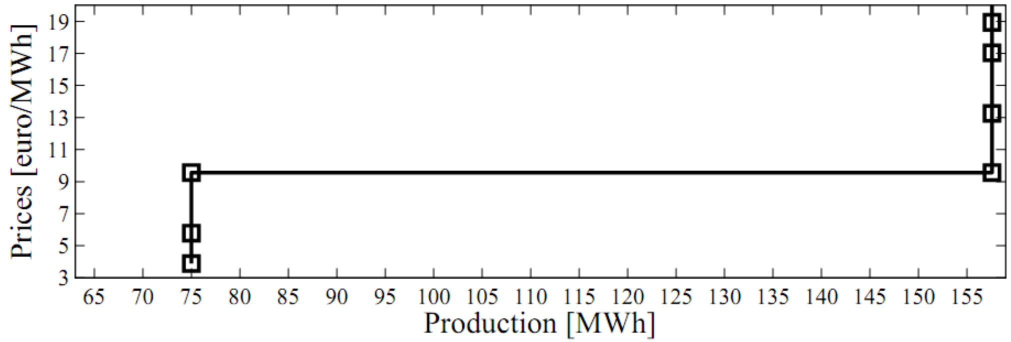


(b) t=9

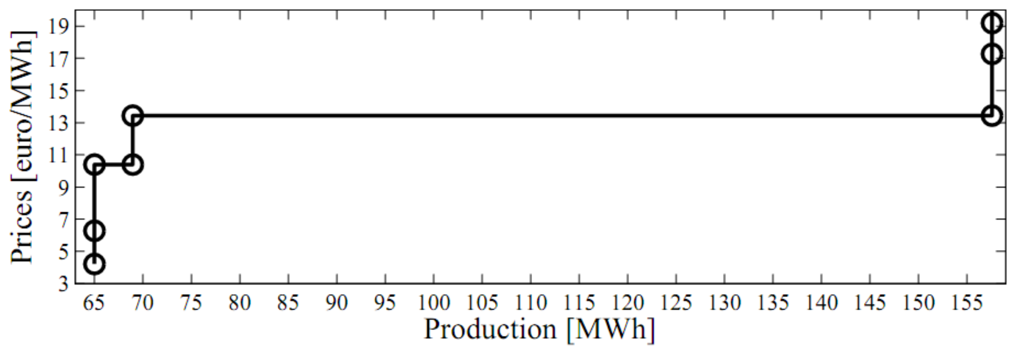
Examples

Six-bus test system→ stochastic model results

Offering curves for strategic generator 1



(c) t=10

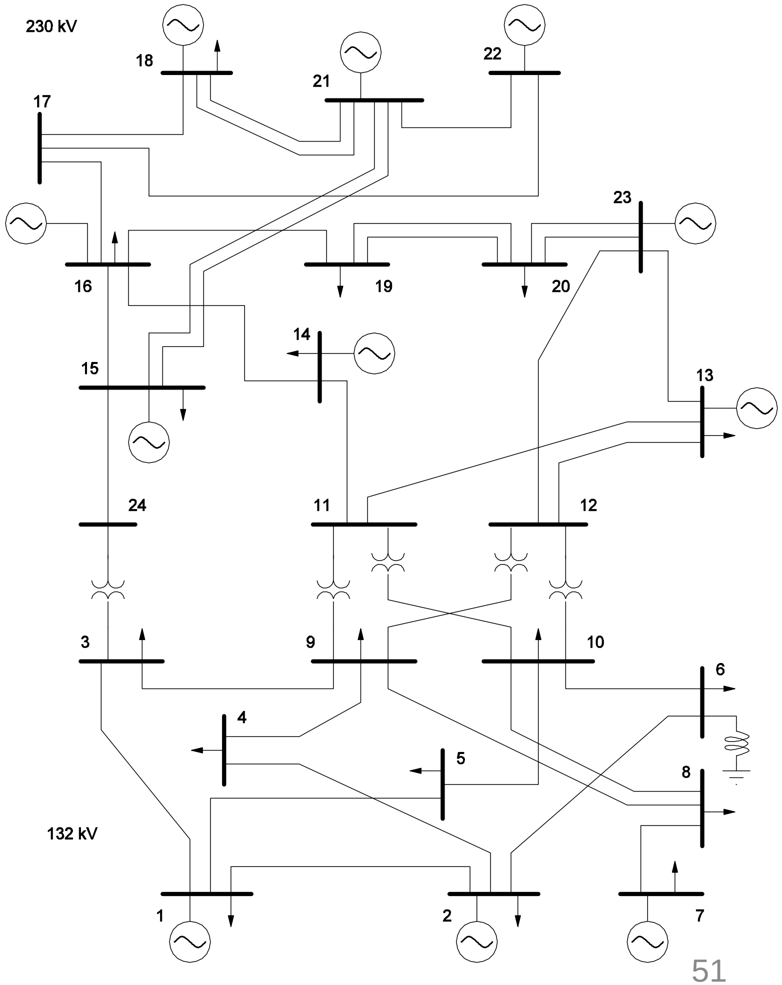


(d) t=13

Examples

IEEE One Area Reliability Test System

- 24 Nodes
- 8 strategic units
- 24 non-strategic units
- 17 consumers
- 24 hours



Examples

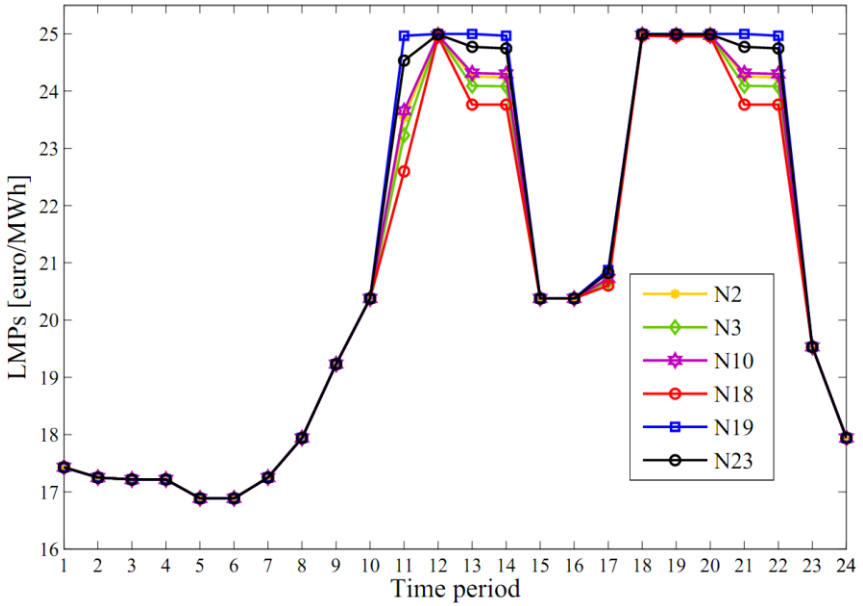
IEEE One Area Reliability Test System → Results

Strategic Offer								
	S1	S2	S3	S4	S5	S6	S7	Total
Production [MWh]	36.48	18.24	8.9416	47.28	31.01	96	37.2	281.13
Profit [€]	27929	13965	3287.3	47959	38689	148120	38983	318932.3
Marginal-Cost Offer								
Production [MWh]	36.48	18.24	10.6	47.28	37.2	96	37.2	283
Profit [€]	27245	13625	3449.3	47296	37815	145170	38773	313373

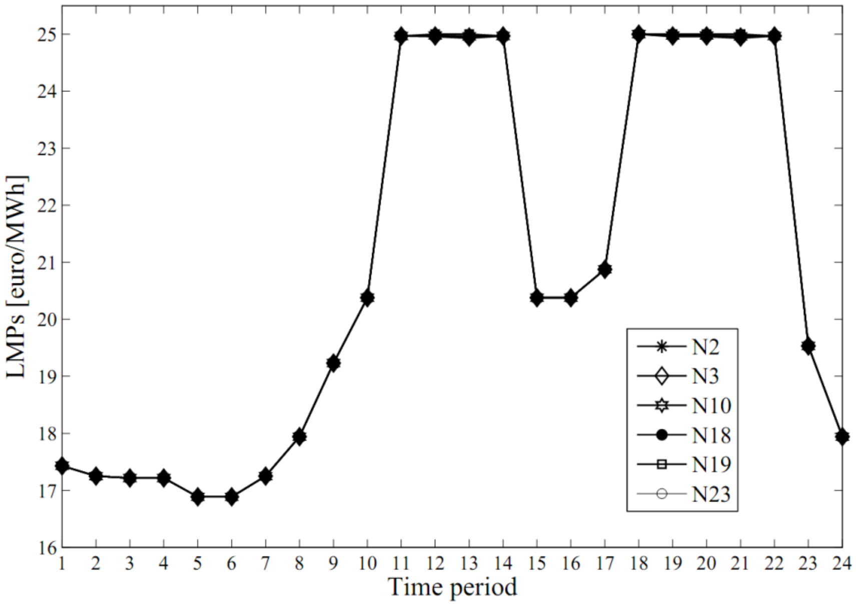
Examples

IEEE One Area Reliability Test System → Results

Marginal cost offer



Strategic offer



Examples

Computational issues

- Model solved using CPLEX 11.0.1 under GAMS on a Sun Fire X4600 M2 with 4 processors at 2.60 GHz and 32 GB of RAM.

Model	6-bus uncongested	6-bus congested	6-bus stochastic	IEEE RTS
CPU Time [s]	2.91	5.82	204.77	449.33

Contents

Conclusions

- Procedure to derive strategic offers for a power producer in a network constrained pool market.
  - LMPs are endogenously generated: MPEC approach.
  - Uncertainty is taken into account.
  - Resulting MILP problem.
- Exercising market power results in higher profit and lower production.
- Network congestion can be used to further increase profit.

Thanks for your attention!

http://www.uclm.es/area/gsee/web/antonio.htm

Appendix A
Computational Issues

- Model has been solved using CPLEX 11.0.1 under GAMS on a Sun Fire X4600 M2 with 4 processors at 2.60 GHz and 32 GB of RAM.
- The computational times are highly dependent on the values of the linearization constants *M*.

Model	6-bus uncongested	6-bus congested	6-bus stochastic	IEEE RTS
CPU Time [s]	2.91	5.82	204.77	449.33

Appendix A
Computational Issues

Heuristic to determine the value of *M:*

1. Solve a (single-level) market clearing considering that all the producers offer at marginal cost.
2. Obtain the marginal value of each relevant constraint.
3. Compute the value of each relevant constant as:

$$M = (\text{dual variable value} + 1) \times 100$$

Appendix B
Stochastic model

$$\underset{\alpha^S_{tibw}, P^S_{tibw} \quad \forall t, \forall i, \forall b, \forall w}{\text{Minimize}}$$

$$\sum_{w} \Pi_w \Big(\sum_{tib} \lambda^S_{tib} P^S_{tibw} - \sum_{t(i \in \Psi_n)b} \beta_{tnw} P^S_{tibw} \Big)$$

subject to:

$$\sum_{b} P^S_{(t+1)ibw} - \sum_{b} P^S_{tibw} \le R^{\mathrm{UP}}_i \qquad \forall t < T, \forall i, \forall w$$

$$\sum_{b} P^S_{tibw} - \sum_{b} P^S_{(t+1)ibw} \le R^{\mathrm{LO}}_i \qquad \forall t < T, \forall i, \forall w$$

$$\beta_{tnw} = \lambda_{tnw} \qquad \forall t, \forall n, \forall w$$

Appendix B
Stochastic model

$$P_{tibw}^{S} \in \arg \left\{ \underset{P_{tibw}^{S}, P_{tjbw}^{O}, P_{tdkw}^{D}}{\text{Minimize}} \quad \sum_{tib} \alpha_{tibw}^{S} P_{tibw}^{S} + \right.$$

$$+ \sum_{tjb} \lambda_{tjbw}^{O} P_{tjbw}^{O} - \sum_{tdk} \lambda_{tdkw}^{D} P_{tdkw}^{D}$$

subject to:

$$\sum_{(i \in \Psi_n)b} P_{tibw}^{S} + \sum_{(j \in \Psi_n)b} P_{tjbw}^{O} - \sum_{(d \in \Psi_n)k} P_{tdkw}^{D} =$$

$$= \sum_{m \in \Theta_n} B_{nm}(\delta_{tnw} - \delta_{tmw}) \quad : \lambda_{tnw} \quad \forall t, \forall n$$

$$0 \leq P_{tibw}^{S} \leq P_{tib}^{S^{\max}} \quad : \mu_{tibw}^{S^{\min}}, \mu_{tibw}^{S^{\max}} \quad \forall t, \forall i, \forall b$$

$$0 \leq P_{tjbw}^{O} \leq P_{tjbw}^{O^{\max}} \quad : \mu_{tjbw}^{O^{\min}}, \mu_{tjbw}^{O^{\max}} \quad \forall t, \forall j, \forall b$$

$$0 \leq P_{tdkw}^{D} \leq P_{tdkw}^{D^{\max}} \quad : \mu_{tdkw}^{D^{\min}}, \mu_{tdkw}^{D^{\max}} \quad \forall t, \forall d, \forall k$$

Appendix B
Stochastic model

$$-C_{nm}^{max} \leq B_{nm}(\delta_{tnw} - \delta_{tmw}) \leq C_{nm}^{\max}$$

$$: \nu_{tnmw}^{\min}, \nu_{tnmw}^{\max} \quad \forall t, \forall n, \forall m \in \Theta_n$$

$$\left.\begin{array}{lll} -\pi \leq \delta_{tnw} \leq \pi & : \xi_{tnw}^{\min}, \xi_{tnw}^{\mathrm{maxw}} & \forall t, \forall n \\ \delta_{tnw} = 0 & : \xi_{tw}^{1} & \forall t, n = 1 \end{array}\right\} \forall w$$

$$\lambda_{tnw} - \lambda_{tnw'} \leq x_{tiww'} M^x \quad \forall t, \forall i \in \Psi_n, \forall w, \forall w' > w$$

$$\lambda_{tnw} - \lambda_{tnw'} \geq (x_{tiww'} - 1) M^x \quad \forall t, \forall i \in \Psi_n, \forall w, \forall w' > w$$

$$\sum_b P_{tibw}^S - \sum_b P_{tibw'}^S \leq y_{tiww'} M^y \quad \forall t, \forall i, \forall w, \forall w' > w$$

$$\sum_b P_{tibw}^S - \sum_b P_{tibw'}^S \geq (y_{tiww'} - 1) M^y \quad \forall t, \forall i, \forall w, \forall w' > w$$

$$x_{tiww'} + y_{tiww'} = 2 z_{tiww'} \quad \forall t, \forall i, \forall w, \forall w' > w$$

$$x_{tiww'}, y_{tiww'}, z_{tiww'} \in \{0, 1\}$$