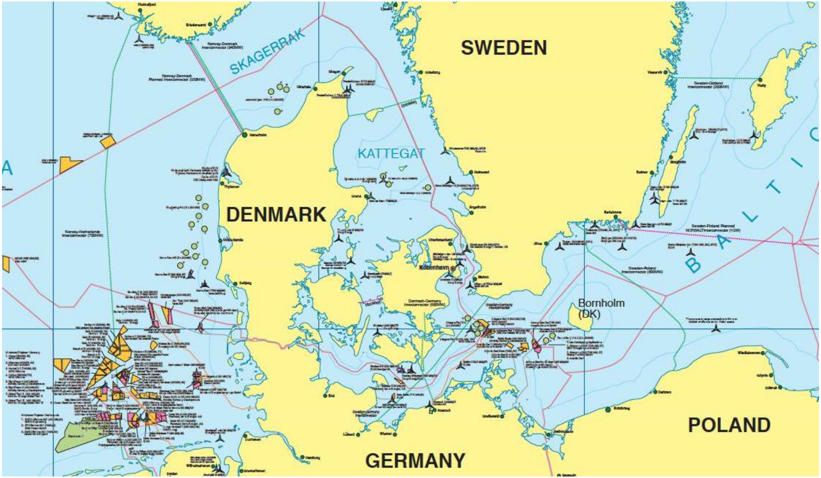
Optimizing Dynamic Power Flow

Anders Rantzer

Automatic Control LTH
Lund Universty

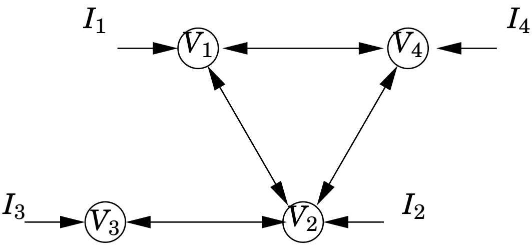


- Combine with water power reservoirs in northern Sweden
- Use wind farms to stabilize network
- AEOLUS project: Distributed coordination of wind turbines

Outline

- **Problem Statements**
- Positive Quadratic Programming
- Optimizing Static Power Flow
- Dynamic Positive Programming
- Optimizing Dynamic Power Flow

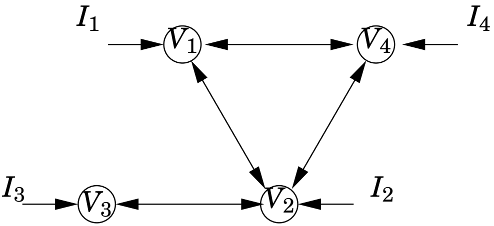
A Power Transmission Network



$$\underbrace{\begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \\ I_4(s) \end{bmatrix}}_{I(s)} = \underbrace{\begin{bmatrix} Y_{12}+Y_{14} & -Y_{12} & 0 & -Y_{14} \\ -Y_{21} & Y_{21}+Y_{23}+Y_{24} & -Y_{23} & -Y_{24} \\ 0 & -Y_{32} & Y_{32} & 0 \\ -Y_{41} & -Y_{42} & 0 & Y_{41}+Y_{42} \end{bmatrix}}_{Y(s)} \underbrace{\begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \\ V_4(s) \end{bmatrix}}_{V(s)}$$

Potential differences drive currents (voltage*current = power)
Price differences drive commodity flows (price*amount = value)

An Optimal Flow Problem for AC Power



$$I_k \in \mathbf{C}$$
$$V_k \in \mathbf{C}$$

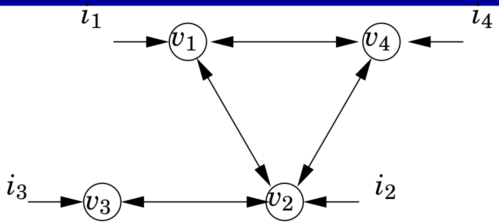
Minimize $\quad$ Re $\sum_k I_k^* V_k$

subject to $I = YV$ and

$$\underline{P}_k \leq \text{Re}\,(I_k^* V_k) \leq \overline{P}_k$$
$$\underline{Q}_k \leq \text{Im}\,(I_k^* V_k) \leq \overline{Q}_k$$
$$\underline{v}_k \leq |V_k| \leq \bar{v}_k \qquad \text{for } k = 1, \dots, 4$$

(Convex relaxation by Lavaei/Low inspired this talk.)

Problem I: Optimizing Static Power Flow



$i_k \in \mathbf{R}$

$v_k \in \mathbf{R}$

Minimize $\sum_k i_k v_k$

subject to $i = Yv$ and

$$i_k v_k \leq \overline{p}_k$$
$$(v_k - v_j)^2 \leq c_{kj}$$
$$\underline{v}_k \leq v_k \leq \overline{v}_k \qquad \text{for all } k, j$$

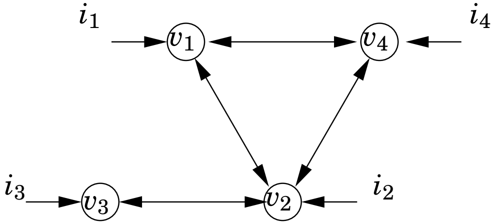
Notice: $\overline{p}_k$ negative at loads, positive at generators.

Motivation: 1) Real DC networks. 2) Approximation of AC.
3) Water tanks. 4) Supply chains

Further questions regarding Problem I

- Are there distributed solution algorithms?
- Will market mechanisms find the optimum?
- Optimize transition when demand changes! (Problem II)

Problem II: Optimizing Dynamic Power Flow



$i_k(t) \in \mathbf{R}$
$v_k(t) \in \mathbf{R}$

Minimize $\sum_k \int_0^\infty i_k(t) v_k(t) dt$

subject to $I(s) = Y(s)V(s)$ and

$$i_k(t)v_k(t) \leq \overline{p}_k$$
$$|v_k(t) - v_j(t)|^2 \leq c_{kj}$$
$$\underline{v}_k \leq v_k(t) \leq \overline{v}_k \qquad \text{for all } k, j$$

Convexly solvable when off-diagonal elements of $Y(s)$ have non-negative impulse response! (e.g. ramp dynamics)

Outline

- Problem Statements
- **Positive Quadratic Programming**
- Optimizing Static Power Flow
- Dynamic Positive Programming
- Optimizing Dynamic Power Flow

Positive Quadratic Programming

Given $A_0, \ldots, A_K \in \mathbf{R}^{n\times n}$ with nonnegative off-diagonal entries and $b_1, \ldots, b_K \in \mathbf{R}$, the following equality holds:

$$\begin{array}{lll} \text{max} & x^T A_0 x \\ \text{subject to} & x \in \mathbf{R}^n_+ \\ & x^T A_k x \geq b_k \\ & k = 1, \ldots, K \end{array} \quad = \quad \begin{array}{ll} \text{max} & \operatorname{trace}(A_0 X) \\ \text{subject to} & X \succeq 0 \\ & \operatorname{trace}(A_k X) \geq b_k \\ & k = 1, \ldots, K \end{array}$$

Proof

If $X = \begin{bmatrix} |x_1|^2 & & * \\ & \ddots & \\ * & & |x_n|^2 \end{bmatrix}$ maximizes the right hand side,

then $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ maximizes the left.

[Kim/Kojima, 2003]

Positive Quadratic Programming

Given $A_0, \ldots, A_K \in \mathbf{R}^{n\times n}$ with nonnegative off-diagonal entries and $b_1, \ldots, b_K \in \mathbf{R}$, the following equality holds:

$$
\begin{array}{lll}
\text{max} & x^T A_0 x & \\
\text{subject to} & x \in \mathbf{R}^n_+ & \\
 & x^T A_k x \geq b_k & \\
 & k = 1, \ldots, K &
\end{array}
=
\begin{array}{ll}
\text{max} & \text{trace}(A_0 X) \\
\text{subject to} & X \succeq 0 \\
 & \text{trace}(A_k X) \geq b_k \\
 & k = 1, \ldots, K
\end{array}
$$

Proof

If $X = \begin{bmatrix} |x_1|^2 & & * \\ & \ddots & \\ * & & |x_n|^2 \end{bmatrix}$ maximizes the right hand side,

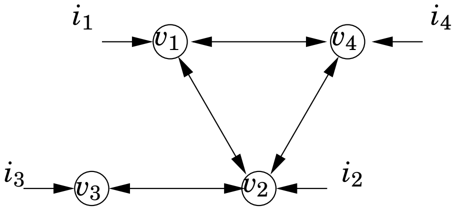
then $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ maximizes the left.

[Kim/Kojima, 2003]

Outline

- Problem Statements
- Positive Quadratic Programming
- **Optimizing Static Power Flow**
- Dynamic Positive Programming
- Optimizing Dynamic Power Flow

An Optimal Flow Problem for DC Power

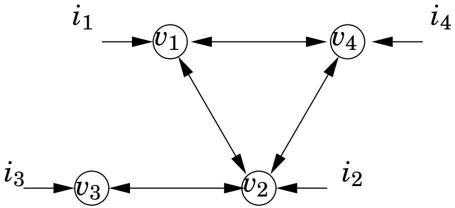


Minimize $\quad i_3v_3 + i_4v_4$

subject to $i = Yv$ and

$$i_1v_1 \leq \overline{p}_1$$
$$i_2v_2 \leq \overline{p}_2$$
$$\underline{v}_k \leq v_k \leq \overline{v}_k \quad \text{for } k = 1, \ldots, 4$$

An Optimal Flow Problem for DC Power



Minimize $\quad (-y_{32}v_2 + y_{32}v_3)v_3 + (-y_{41}v_1 - y_{42}v_2 + y_{41}v_4 + y_{42}v_4)v_4$

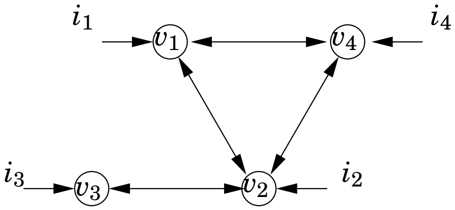
subject to $\quad (y_{12}v_1 + y_{14}v_1 - y_{12}v_2 - y_{14}v_4)v_1 \leq \overline{p}_1$

$$(-y_{21}v_1 + y_{21}v_2 + y_{23}v_2 + y_{24}v_2 - y_{23}v_3 - y_{24}v_4)v_2 \leq \overline{p}_2$$

$$|\underline{v}_k|^2 \leq |v_k|^2 \leq |\overline{v}_k|^2$$

Note: The problem is convex in $|v_1|^2, \ldots, |v_4|^2$!

An Optimal Flow Problem for DC Power



$$\begin{aligned}
\text{Minimize} \quad & (-y_{32}v_2 + y_{32}v_3)v_3 + (-y_{41}v_1 - y_{42}v_2 + y_{41}v_4 + y_{42}v_4)v_4 \\
\text{subject to} \quad & (y_{12}v_1 + y_{14}v_1 - y_{12}v_2 - y_{14}v_4)v_1 \le \overline{p}_1 \\
& (-y_{21}v_1 + y_{21}v_2 + y_{23}v_2 + y_{24}v_2 - y_{23}v_3 - y_{24}v_4)v_2 \le \overline{p}_2 \\
& |\underline{v}_k|^2 \le |v_k|^2 \le |\overline{v}_k|^2
\end{aligned}$$

Note: The problem is convex in $|v_1|^2, \ldots, |v_4|^2$!

Dual Positive Quadratic Programming

Given $A_0, \dots, A_K \in \mathbf{R}^{n\times n}$ with nonnegative off-diagonal entries and $b_1, \dots, b_K \in \mathbf{R}$, the following equality holds:

$$\begin{array}{lll} \text{max} & x^T A_0 x & = & \text{min} & -\sum_k \lambda_k b_k \\ \text{subject to} & x \in \mathbf{R}^n_+ & & \text{subject to} & \lambda_1, \dots, \lambda_K \geq 0 \\ & x^T A_k x \geq b_k & & & 0 \succeq A_0 + \sum_k \lambda_k A_k \\ & k = 1, \dots, K & & & \end{array}$$

Interpretation:

In the power flow example, λ_k is the price of power at node k.

Dual Positive Quadratic Programming

Given $A_0, \ldots, A_K \in \mathbf{R}^{n \times n}$ with nonnegative off-diagonal entries and $b_1, \ldots, b_K \in \mathbf{R}$, the following equality holds:

$$
\begin{array}{llcll}
\text{max} & x^T A_0 x & = & \text{min} & -\sum_k \lambda_k b_k \\
\text{subject to} & x \in \mathbf{R}^n_+ & & \text{subject to} & \lambda_1, \ldots, \lambda_K \geq 0 \\
 & x^T A_k x \geq b_k & & & 0 \succeq A_0 + \sum_k \lambda_k A_k \\
 & k = 1, \ldots, K & & &
\end{array}
$$

Interpretation:

In the power flow example, λ_k is the price of power at node k.

Dual Positive Quadratic Programming

Given $A_0, \dots, A_K \in \mathbf{R}^{n \times n}$ with nonnegative off-diagonal entries and $b_1, \dots, b_K \in \mathbf{R}$, the following equality holds:

$$
\begin{array}{lllll}
\text{max} & x^T A_0 x & = & \text{min} & -\sum_k \lambda_k b_k \\
\text{subject to} & x \in \mathbf{R}^n_+ & & \text{subject to} & \lambda_1, \dots, \lambda_K \geq 0 \\
 & x^T A_k x \ \geq \ b_k & & & 0 \succeq A_0 + \sum_k \lambda_k A_k \\
 & k = 1, \dots, K & & &
\end{array}
$$

Distributed solution:

The agent at node k bying power over node jk compares prices at both ends and adjusts for power losses in the link.

Outline

- Problem Statements
- Positive Quadratic Programming
- Optimizing Static Power Flow
- **Dynamic Positive Programming**
- Optimizing Dynamic Power Flow

Positive systems have nonnegative impulse response

If the matrices A, B, C and D have nonnegative coefficients except for the diagonal of A, then the system

$$\begin{aligned} \frac{dx}{dt} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

has non-negative impulse response.

Example:

$$\begin{aligned} L\frac{di}{dt} &= -Ri + v \\ y &= i \end{aligned}$$

inductive transmission line

Positive systems have nonnegative impulse response

If the matrices A, B, C and D have nonnegative coefficients except for the diagonal of A, then the system

$$\begin{aligned} \frac{dx}{dt} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

has non-negative impulse response.

Example:

$$\begin{aligned} \frac{dv}{dt} &= -\alpha v + u \\ y &= v \end{aligned}$$

generator ramp dynamics

Positive systems

Suppose the matrices A, B, C and D have nonnegative coefficients except for the diagonal of A:

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

Properties:

- Stability verified by linear or diagonal Lyapunov functions.
- Maximal gain for zero frequency:

$$\max_{\omega} \|C(i\omega I - A)^{-1}B + D\| = \|D - CA^{-1}B\|$$

Dynamic Positive Programming

Let $A_0(s),\dots,A_K(s)$ have off-diagonal entries with nonnegative impulse response and $b_1,\dots,b_K \in \mathbf{R}$. Then the following equality holds:

$$
\begin{array}{ll}
\text{max} & \int_{-\infty}^{\infty} x(i\omega)^* A_0(i\omega) x(i\omega) d\omega \\
\text{subject to} & \int_{-\infty}^{\infty} x(i\omega)^* A_k(i\omega) x(i\omega) d\omega \ \geq \ b_k \\
 & x \in \mathbf{H}_+^n, k = 1,\dots,K
\end{array}
$$

$$
= \quad \begin{array}{ll}
\text{max} & \int_{-\infty}^{\infty} \text{trace}(A_0 X) d\omega \\
\text{subject to} & \int_{-\infty}^{\infty} \text{trace}(A_k X) d\omega \geq b_k \\
 & X(i\omega) \succeq 0, k = 1,\dots,K
\end{array}
$$

where $\mathbf{H}_+^n$ consists of all stable transfer functions with nonnegative impulse response.

Positive Quadratic Programming

Let $A_0(s), \ldots, A_K(s)$ have off-diagonal entries with nonnegative impulse response and $b_1, \ldots, b_K \in \mathbf{R}$. Then the following equality holds:

$$\begin{array}{llcll} \text{max} & \int_{-\infty}^{\infty} x^* A_0 x d\omega & = & \text{max} & \int_{-\infty}^{\infty} \text{trace}(A_0 X) d\omega \\ \text{subject to} & x \in \mathbf{H}_+^n & & \text{subject to} & X \succeq 0 \\ & \int_{-\infty}^{\infty} x^* A_k x d\omega \geq b_k & & & \int_{-\infty}^{\infty} \text{trace}(A_k X) d\omega \geq b_k \\ & k = 1, \ldots, K & & & k = 1, \ldots, K \end{array}$$

Proof

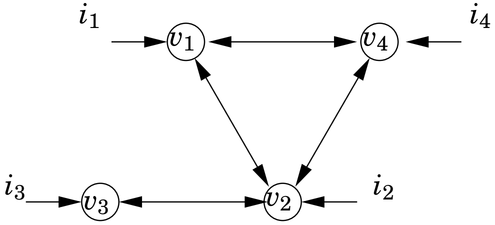
If $X = \begin{bmatrix} |x_1|^2 & & * \\ & \ddots & \\ * & & |x_n|^2 \end{bmatrix}$ maximizes the right hand side,

then $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ maximizes the left.

Outline

- Problem Statements
- Positive Quadratic Programming
- Optimizing Static Power Flow
- Dynamic Positive Programming
- **Optimizing Dynamic Power Flow**

Problem II: Optimizing Dynamic Power Flow



$i_k \in \mathbf{R}$

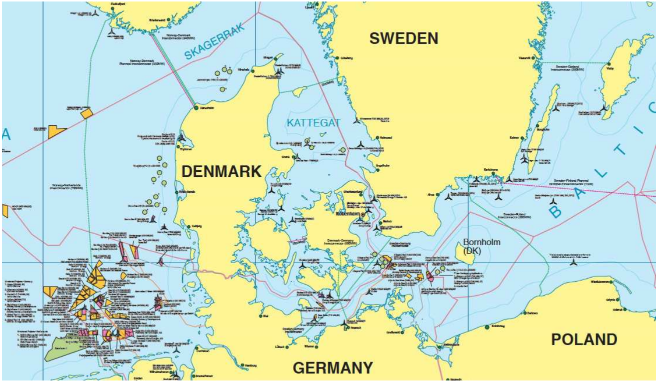
$v_k \in \mathbf{R}$

Minimize $\sum_k \int_0^\infty i_k(t) v_k(t) dt$

subject to $I(s) = Y(s)V(s)$ and
$$i_k(t)v_k(t) \leq \overline{p}_k$$
$$|v_k(t) - v_j(t)|^2 \leq c_{kj}$$
$$\underline{v}_k \leq v_k(t) \leq \overline{v}_k \qquad \text{for all } k, j$$

Convexly solvable when off-diagonal elements of $Y(s)$ have non-negative impulse response! (Inductive loads)

Summary



- Positive Quadratic Programming
- Optimizing Static Power Flow
- Dynamic Positive Programming
- Optimizing Dynamic Power Flow