Optimal Radio-Mode Switching for wireless Networked Control Systems

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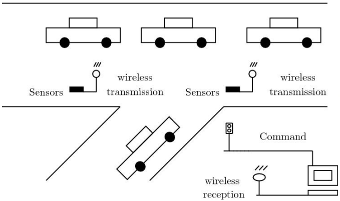
Material from:

- Energy-aware wireless networked control using radio-mode management, N. Cardoso, Ph.D. Dissertation, University of Grenoble, Oct. 2012.
- Energy-aware wireless networked control using radio-mode management, N. Cardoso de Castro, C. Canudas-de-Wit, and F. Garin. ACC 2012, Montréal, Canada
- Smart Energy-Aware Sensors for Event-Based Control, N. Cardoso De Castro; D. E. Quevedo; F. Garin; C. Canudas-de-Wit. IEEE CDC'12

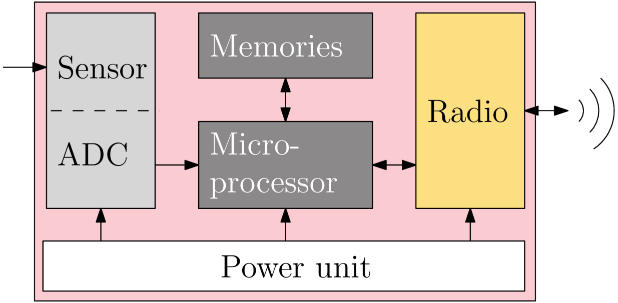
Motivation

1. Sensors will be packaged together with communication protocols, RF electronics, and energy management systems.
2. Constraints: low cost, ease of replacement, low energy consumption, and efficient communication links.
3. Implications: intelligent sensors with low consumption (sleep and wake-up modes), for life-time maximization



Example: Traffic system with distributed density sensors. Traffic flow sensor

The smart sensor wireless node



Radio is often the main energy-consumer

Executing 3 million instructions is equivalent to transmitting 1000 bits at a distance of 100 meters in terms of expended energy

Physical layer

Power Control

- Transmission power is related to communication **reliability**
- Power control aims to **save energy**, limit interferences, face channel varying conditions

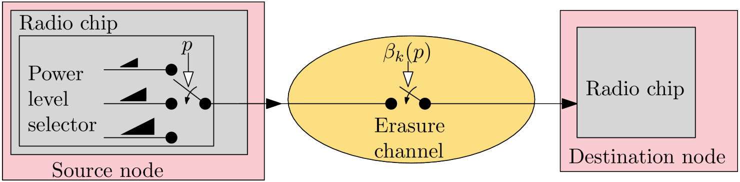


Figure: A source can adapt its transmission power level to change the success probability of the transmission.

Data Link (MAC) layer

Radio-mode management

- Radio-mode = **state of activity** of the radio chip (*e.g.* `Tx`, `Rx`, `Idle`, `Sleep`) where some components are turned off
- Control community only considers `ON` and `OFF`

- θ_i–Energy stay cost per unit of time (at node i),
- $\theta_{i,j}$–Energy transition costs between i and j.

Choosing a mode is a **trade-off** between **energy** consumption and node **awareness**.

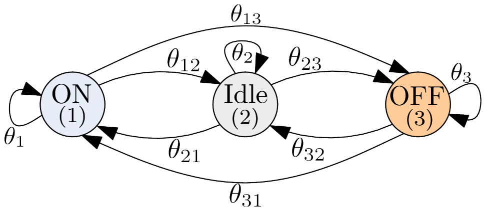


Figure: Illustration of a 3 radio-modes switching automata

Data Link (MAC) layer (cont.)

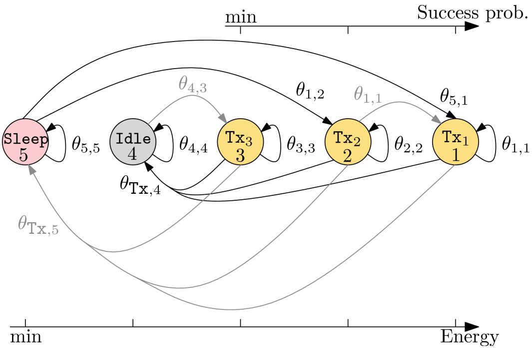
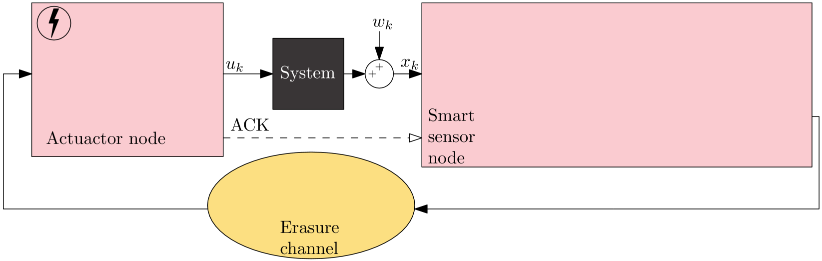


Figure: Illustration of a 5 radio-modes switching automata

- Low-consuming radio-mode not used in control
- Higher power modes have higher probability of transmission success
- **Problem considered here**: co-design of mode management and control laws to save further energy

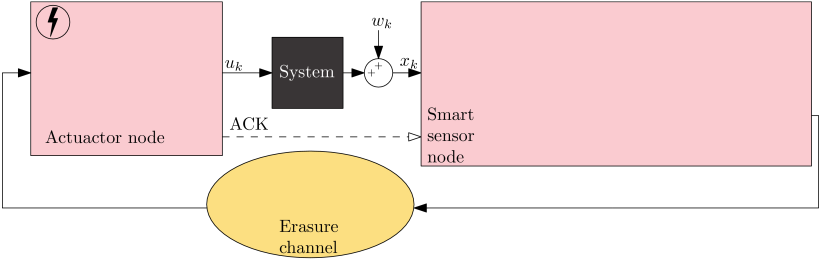
Model and setup



- 2 nodes scenario
- Battery-powered smart sensor node (with computation capabilities)
- Energy saving at the sensor side
- Time-triggered sensing (negligible cost) and Event-Triggered transmission

Problem: How to design the radio mode, and the control input u_k

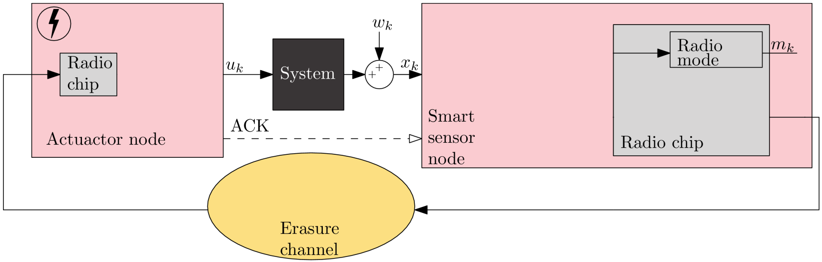
Model and setup



$$x_{k+1} = Ax_k + Bu_k + w_k$$

$$x_k \in \mathbb{R}^{n_x}, u_k \in \mathbb{R}^{n_u}$$

Model and setup



$$x_{k+1} = Ax_k + Bu_k + w_k$$

$$m_k \in \mathbb{M} \triangleq \mathbb{M}_1 \cup \mathbb{M}_2$$

$$\mathbb{M}_1 \triangleq \{1, 2, \cdots, N_1\}$$

$$\mathbb{M}_2 \triangleq \{N_1 + 1, N_1 + 2, \cdots, N\}$$

$$\theta_{i,j} - \text{Transition cost}, \forall(i,j) \in \mathbb{M}$$

$$\theta_i - \text{Stay cost}, \forall(i) \in \mathbb{M}$$

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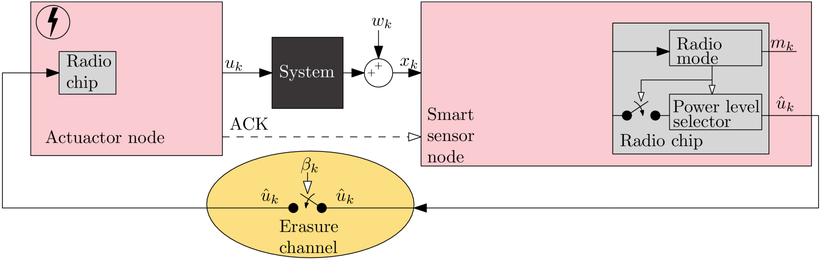

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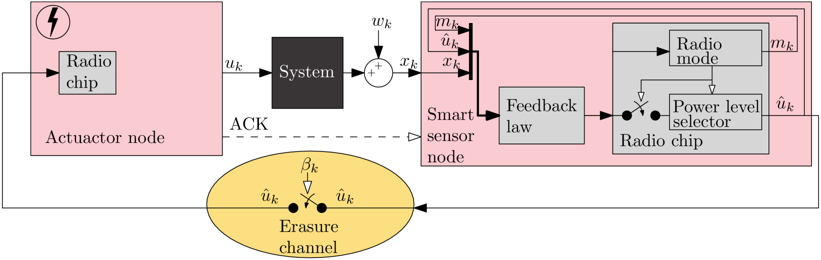
Model and setup



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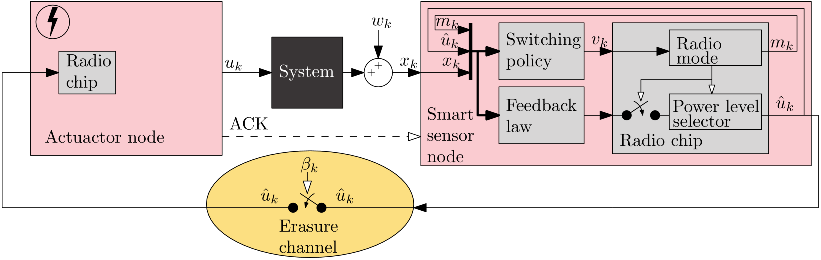

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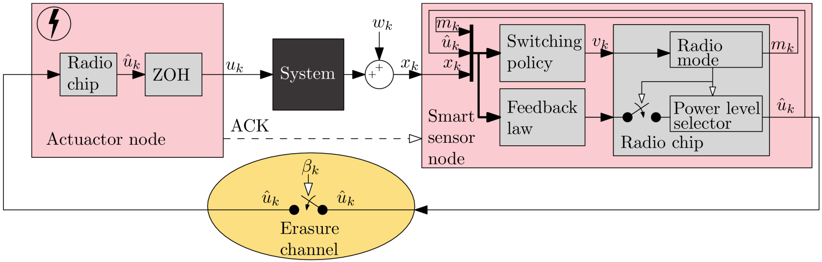
$$v_k = \eta(x_k, u_{k-1}, m_k)$$

Model and setup



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$$\hat{u}_k = \mu(x_k, u_{k-1}, m_k)$$
$$v_k = \eta(x_k, u_{k-1}, m_k)$$
$$u_k = \begin{cases} \beta_k \hat{u}_k + (1-\beta_k)u_{k-1}, & \text{if Tx,} \\ u_{k-1}, & \text{otherwise.} \end{cases}$$

Switched model

$\forall k$: choice between N radio-modes $\Rightarrow$ N subsystems
Switching is triggered by the switching decision v_k

Given μ and η:

$$\begin{cases} z_{k+1} = f_{v_k}(z_k, \hat{u}_k, \beta_k, \omega_k) \\ m_{k+1} = v_k = \eta(z_k, m_k) \\ \hat{u}_k = \mu(z_k, m_k), \end{cases}$$

$$f_{v_k}(z_k, \hat{u}_k, \beta_k, \omega_k) = \Phi_{v_k}(\beta_k) z_k + \Gamma_{v_k}(\beta_k)\hat{u}_k + \omega_k$$

$\tilde{u}_k = u_{k-1}$ (control memory)

$z_k = \begin{bmatrix} x_k \\ \tilde{u}_k \end{bmatrix}$ (augmented state)

$(z_k, m_k) \in \mathbb{X} = \mathbb{R}^{n_x + n_u} \times \mathbb{M}$ (switched system state)

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If $v \in \mathbb{M}_1$ (Tx case):

$$\Phi_{v_k}(\beta_k) = \begin{cases} \Phi_{CL} = \begin{bmatrix} A & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} & \text{if } \beta_k = 1 \\ \Phi_{OL} = \begin{bmatrix} A & B \\ \mathbf{0} & \mathbf{I} \end{bmatrix} & \text{if } \beta_k = 0. \end{cases}$$

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If $v \in \mathbb{M}_2$ (no Tx case):

$\Phi_{v_k}(\beta_k) = \Phi_{OL} \; \forall \beta_k$

$\Gamma_{v_k}(\beta_k) = \Gamma_{OL} \; \forall \beta_k$

$\tilde{u}_k = u_{k-1}$ (control memory)

$z_k = \begin{bmatrix} x_k \\ \tilde{u}_k \end{bmatrix}$ (augmented state)

$(z_k, m_k) \in \mathbb{X} = \mathbb{R}^{n_x+n_u} \times \mathbb{M}$ (switched system state)

Optimisation problem

Switched formulation of the cost-to-go

$$\ell_{v_k}(z_k, m_k, \hat{u}_k, \beta_k) = z_k^\top Q_{v_k}(\beta_k) z_k + \hat{u}_k^\top R_{v_k}(\beta_k) \hat{u}_k + \underbrace{\theta_{m_k, v_k}}_{\text{transmission energy}}$$

If $v_k \in \mathbb{M}_1$, **(Tx case):**
$$Q_{v_k}(\beta_k) = \begin{cases} Q_{CL} = \begin{bmatrix} \bar{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} & \text{if } \beta_k = 1 \\ Q_{OL} = \begin{bmatrix} \bar{Q} & \mathbf{0} \\ \mathbf{0} & \bar{R} \end{bmatrix} & \text{if } \beta_k = 0 \end{cases}$$

$$R_{v_k}(\beta_k) = \begin{cases} R_{CL} = \bar{R} & \text{if } \beta_k = 1 \\ R_{OL} = \mathbf{0} & \text{if } \beta_k = 0 \end{cases}$$

If $v_k \in \mathbb{M}_2$, **(no Tx case):**
$$Q_{v_k}(\beta_k) = Q_{OL} \ \forall \beta_k$$
$$R_{v_k}(\beta_k) = R_{OL} \ \forall \beta_k$$

Optimisation problem (cont.)

Finite horizon cost function from 0 to H

$$J_{\mathcal{U},\mathcal{V}}(z_0, m_0) = \underset{\substack{\beta_k,\, \omega_k \\ k = 0, 1, \ldots, H-1}}{\mathrm{E}} \left[\ell_F(z_H, m_H) + \sum_{i=0}^{H-1} \lambda^k \ell_{v_i}(z_i, m_i, \hat{u}_i, \beta_i) \right]$$

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$\hat{u}_k = \mu(z_k, m_k)$
$\mathcal{U} = \{u_0, u_1, \ldots, u_{H-1}\}$: control sequence
$\mathcal{V} = \{v_0, v_1, \ldots, v_{H-1}\}$: switching sequence

z_0, m_0: initial condition
H: horizon length
λ: discount factor
$\ell_F(z, m)$: final cost

Optimisation problem (cont.)

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Infinite horizon $H \to \infty$

- $\lambda < 1$
- Final cost $\ell_F(z,m) = 0$
- optimal stationary feedback $u^* = \mu(z,m)$ and $v^* = \eta(z,m)$ independent of k
- $J^*(z_0,m_0) \triangleq \min_{\mu,\eta} J_{\mu,\eta}(z_0,m_0)$

Optimisation problem (cont.)

Finite horizon cost function from 0 to H

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Finite receding horizon from k to $k + H - 1$

- $\lambda = 1$
- optimal stationary feedback: $u^*_{[k,k+H-1]}(z, m)$ and $v^*_{[k,k+H-1]}(z, m)$
- Finite-Time implementation $\ell_F(z_H, m_H)$
- $J^*(z_k, m_k) \triangleq \min_{\mathcal{U},\mathcal{V}} J_{\mathcal{U},\mathcal{V}}(z_k, m_k)$

Dynamic Programming

Bellman's Principle of Optimality

$J^*_{H-i}(z,m)$ is the optimal cost function, $k = H - i$ over an horizon i:

$$J^*_{H-i}(z,m) = \min_{(\hat{u},v)\in\mathbb{U}(z)} \left\{ \mathop{\mathrm{E}}_{\beta,\omega} \left[\lambda^{H-i}\ell_v(z,m,\hat{u},\beta) + J^*_{H-i+1}(f_v(z,\hat{u},\omega,\beta),v) \right] \right\}$$

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The Value Iteration method

Use previous recursing to compute the optimal cost backward in time, with $V_0(z,m) \triangleq 0$

$$V_{i+1}(z,m) = \min_{(\hat{u},v)\in\mathbb{U}(z)} \left\{ \underset{\beta,\omega}{\mathrm{E}} \left[\ell_v(z,m,\hat{u},\beta) + \lambda V_i(f_v(z,\hat{u},\omega,\beta),v) \right] \right\}$$

$$(\mu^*_i(z,m),\eta^*_i(z,m)) \triangleq \arg\min_{(\hat{u},v)\in\mathbb{U}(z)} \left\{ \underset{\beta,\omega}{\mathrm{E}} \left[\ell_v(z,m,\hat{u},\beta) + \lambda V_i(f_v(z,\hat{u},\omega,\beta),v) \right] \right\}$$

$$\lim_{i\to\infty} V_i(z,m) = J^*(z,m)$$

Implementation issues

Offline computation

$\bar{\mathbb{X}}$ = discretise($\mathbb{X}$);
$V_0(z,m) = 0$;
for(i = 1 to i = I){
  compute $V_{i+1}(z,m) = \mathcal{F}(V_i(z,m))$
    $\forall\ (z,m) \in \bar{\mathbb{X}}$;
}

I is large enough to assume convergence

Provides the optimal Value Function $V_\infty(z,m)$, and an optimal joint policy μ^*, η^*, along the set $\bar{\mathbb{X}}$, i.e.

$$v^* = \eta^*(z,m), \quad u^* = \mu^*(z,m)$$

Online computation

At each time k:
get (z_k, m_k);
if($(z_k, m_k) \in \bar{\mathbb{X}}$){
  compute $v_k^* = \eta^*(z_k, m_k)$;
}else{
  set $v_k^* = 1$;
}
set mode m_{k+1} to v_k^*;
if($v_k^* \in \mathbb{M}_1$){
  compute $u_k^* = \mu^*(z_k, m_k)$;
  send update u_k^*;
}

Infinite horizon solution - Deterministic case

Deterministic case

- No noise ($w_k = 0 \ \forall \ k > 0$),
- No dropout ($\beta_k = 1 \ \forall \ k > 0$)
  $\Rightarrow$ only one transmitting mode ($N_1 = 1$)

An explicit formulation of the value function $V_i(z, m)$ can be computed.

Idea

Exploit a structure of $V_i(z, m)$ preserved along iterations.

$$V_i(z, m) = \min_{(\Pi, \boldsymbol{\pi}) \in \mathcal{P}_i} \left\{ z^\top \Pi z + \pi_m \right\}$$

Compute $\mathcal{P}_i$ rather than $V_i(z, m)$.
$(\Pi, \boldsymbol{\pi})$– set of matrices and vectors.

Infinite horizon solution - Deterministic case–details

$$V_i(z,m) = \min_{(\Pi,\boldsymbol{\pi})\in\mathcal{P}_i} \left\{ z^\top \Pi z + \pi_m \right\}$$

Compute $\mathcal{P}_i$ rather than $V_i(z,m)$.

$\mathcal{P}_i = \{(\Pi_1,\boldsymbol{\pi}_1),(\Pi_2,\boldsymbol{\pi}_2),\ldots\}$, $\text{card}(\mathcal{P}_i) = 2^i$
Given $(\Pi,\boldsymbol{\pi})$: Π is a symmetric matrix and $\boldsymbol{\pi} = \left[\pi_1,\ldots,\pi_N\right] \in \mathbb{R}^N$ is a vector.

$$\mathcal{P}_1 = \left\{ \left(\mathbf{0}, \left[0, \quad 0, \quad \ldots, \quad 0\right]\right)\right\}$$

$$\mathcal{P}_{i+1} = \mathcal{P}_{i+1}^{(1)} \cup \mathcal{P}_{i+1}^{(2)}$$

$$\mathcal{P}_{i+1}^{(1)} \triangleq \Big\{ \big(Q_{CL} + \lambda\Phi_{CL}^\top \Pi \Phi_{CL} - \lambda\kappa_\Pi^\top \Gamma_{CL}^\top \Pi \Phi_{CL}, \left[(\theta_{1,1}+\lambda\pi_1), \quad \ldots, \quad (\theta_{N,1}+\lambda\pi_1)\right]\big)$$
$$\text{such that } (\Pi,\boldsymbol{\pi}) \in \mathcal{P}_i \text{ and } \kappa_\Pi = (R_{CL} + \lambda\Gamma_{CL}^\top \Pi \Gamma_{CL})^{-1}\lambda\Gamma_{CL}^\top \Pi \Phi_{CL} \Big\}$$

$$\mathcal{P}_{i+1}^{(2)} \triangleq \left\{ \left(Q_{OL} + \lambda\Phi_{OL}^\top \Pi \Phi_{OL}, \begin{bmatrix} \min_{v\in\mathbb{M}_2} \{\theta_{1,v} + \lambda\pi_v\} \\ \vdots \\ \min_{v\in\mathbb{M}_2} \{\theta_{N,v} + \lambda\pi_v\} \end{bmatrix}^\top \right) \text{such that } (\Pi,\boldsymbol{\pi}) \in \mathcal{P}_i \right\}$$

Simulation results (Offline)

Scalar linear unstable system, with $N = 3$.
Static state feedback $\mu(x_k) = -Kx_k$
K is given

The switching policy: $\eta^*(x_k, u_{k-1}, m_k)$
Black $\Rightarrow$ "go to mode 3 (Sleep)".
Dark gray $\Rightarrow$ 'go to mode 2 (Idle)".
Light gray $\Rightarrow$ "go to mode 1 (Tx)".

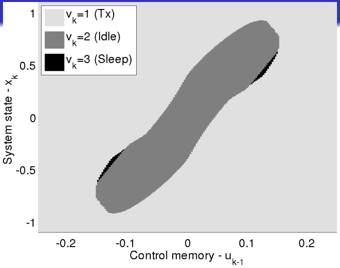

Figure: $m_k = 1$ (Tx).

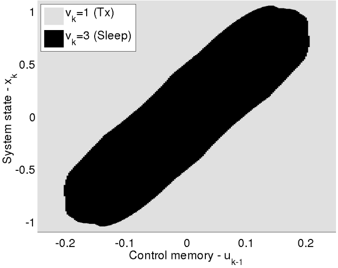

Figure: $m_k = 2$ (Idle).

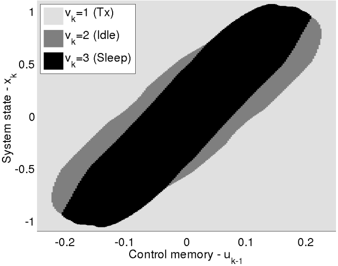

Figure: $m_k = 3$ (Sleep).

Simulation results (Online)

Comparison to periodic switching patterns, $\epsilon = 0.3$ (30% of messages are dropped):

- Blue: Event-based Optimal trajectory
- Red: Best-optimal periodic trajectory (2ON, 2Idle, 2OFF)
- drops: Bernoulli distribution (same in both cases)

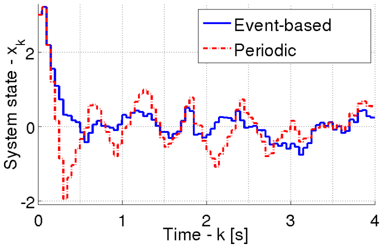

Figure: Output of the system, x_k.

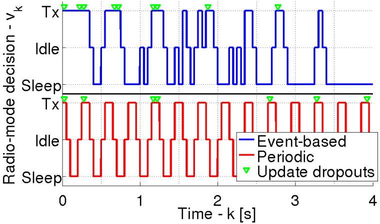

Figure: Switching decision, v_k.

Finite horizon case

Optimisation problem (Finite receding horizon):

$$\begin{cases} z_{k+1} = f_{v_k}(z_k, \hat{u}_k, \beta_k, \omega_k) \\ m_{k+1} = v_k = \eta(z_k, m_k), \quad \hat{u}_k = \mu(z_k, m_k), \end{cases}$$

Find $(\mathcal{U}^*, \mathcal{V}^*)$ such that

$$J_{\mathcal{U}^*,\mathcal{V}^*}(z_k, m_k) = \min_{\mathcal{U},\mathcal{V}} \left\{ \underset{\substack{\beta_k,\, \omega_k \\ k=0,1,\ldots}}{\mathrm{E}} \left[z_{k+H}^\top Q_F z_{k+H} + \sum_{i=k}^{k+H-1} \ell_{v_i}(z_i, m_i, \hat{u}_i, \beta_i) \right] \right\}$$

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Optimisation problem (Finite receding horizon):

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Solution of the optimisation problem are the stationary functions:

$\forall i = 1 \ldots H$,

$$\mathcal{U}^*(z, m) = \{u_k^*, u_{k+1}^*, \ldots, u_{k+H-1}^*\}, \quad u_{k+i-1}^* = \mu_i^*(z, m),$$

$$\mathcal{V}^*(z, m) = \{v_k^*, v_{k+1}^*, \ldots, v_{k+H-1}^*\}, \quad v_{k+i-1}^* = \eta_i^*(z, m),$$

Only the first function is applied: $u_k^* = \mu_1^*(z_k, m_k)$, $v_k^* = \eta_1^*(z_k, m_k)$

Input-to-State practical stability

Assumptions

- Deterministic case, and no drops
- There exists $\kappa \in \mathbb{R}^{n_u \times (n_x + n_u)}$, and $Q_F > 0$ such that:

$$(\Phi_{CL} - \Gamma_{CL}\kappa)^\top Q_F (\Phi_{CL} - \Gamma_{CL}\kappa) - Q_F + Q_{CL} + \kappa^\top R_{CL} \kappa \leq 0$$
$$\text{and} \quad \max\{|\operatorname{eigs}(\Phi_{CL} - \Gamma_{CL}\kappa)|\} \leq 1.$$

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Theorem

The closed-loop system admits a GISpS-Lyapunov function, and then it is GISpS, *i.e.* there exist a $\mathcal{KL}$-function γ, and a constant $c \geq 0$, such that, for all $(z_0, m_0) \in \mathbb{X}$:

$$\|z_k\| \leq \gamma(\|z_0\|, k) + c, \quad k \in \mathbb{Z}_{\geq 0}$$

Proof. The GISpS-Lyapunov function is:

$$V_i(z, m) = \min_{(\Pi, \boldsymbol{\pi}) \in \mathcal{P}_i} \left\{ z^\top \Pi z + \pi_m \right\}$$

Simulation results

Second order linear unstable system, with $N = 3$.
Static state feedback $\mu(x_k) = -Kx_k$

2 modes vs. 3 modes comparison

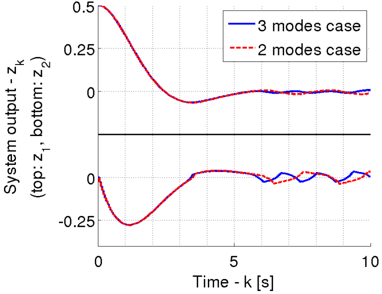

Figure: Output of the system x_k

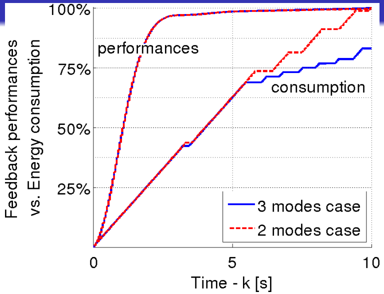

Figure: Perfomance vs. Consumption

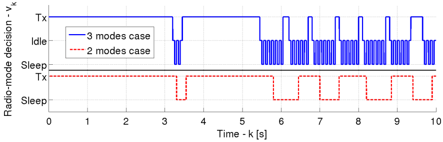

Figure: Switching decision v_k

Conclusion

Summary:

- Formulation of control problem accounting for several radio modes
- Solutions via Dynamic Programming
- Convergence of the Value Iteration method in the infinite/finite case
- Stability assessment in the deterministic finite case

More to do:

- Stability in the stochastic finite case
- Relax optimality to lighten the computation burden
- Stability in the infinite case
- Extension to multi-node setup