An algorithm for cooperative calibration of cameras networks

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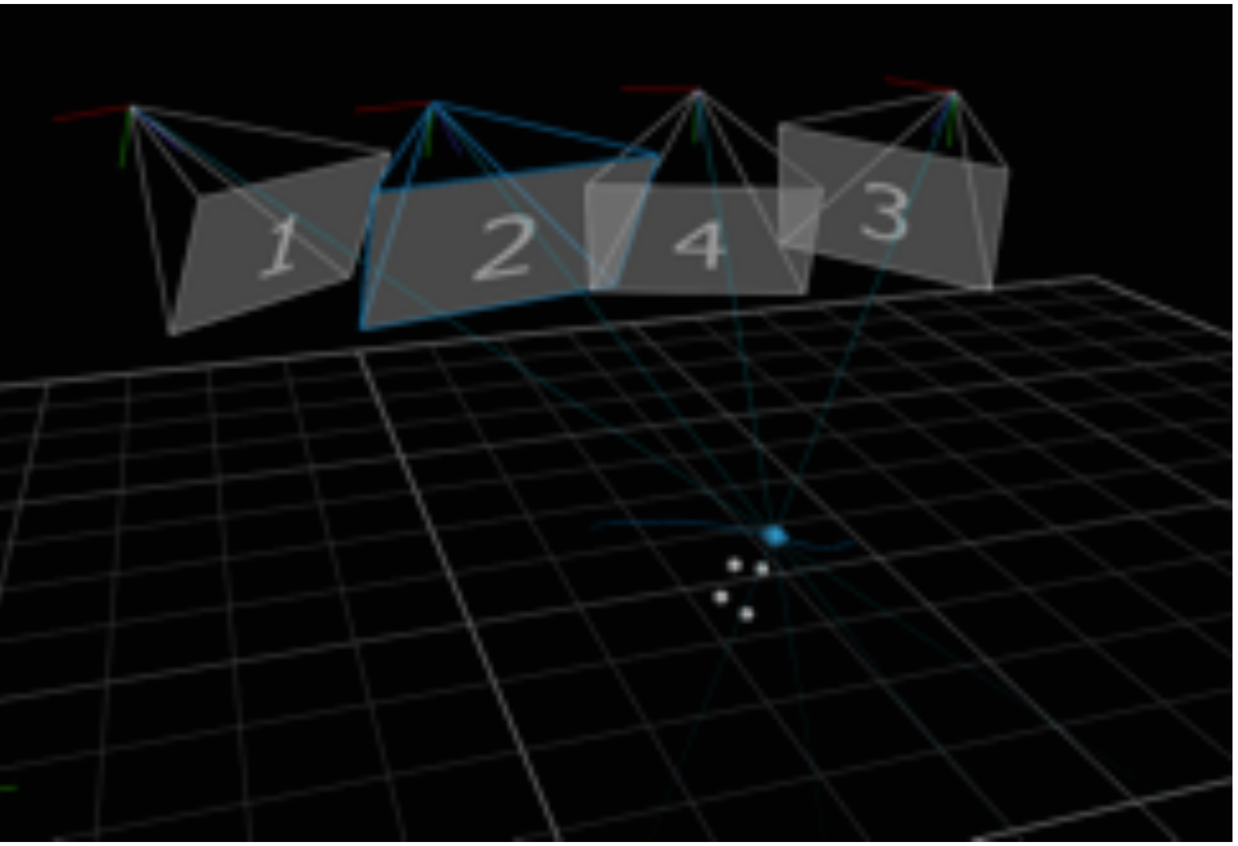
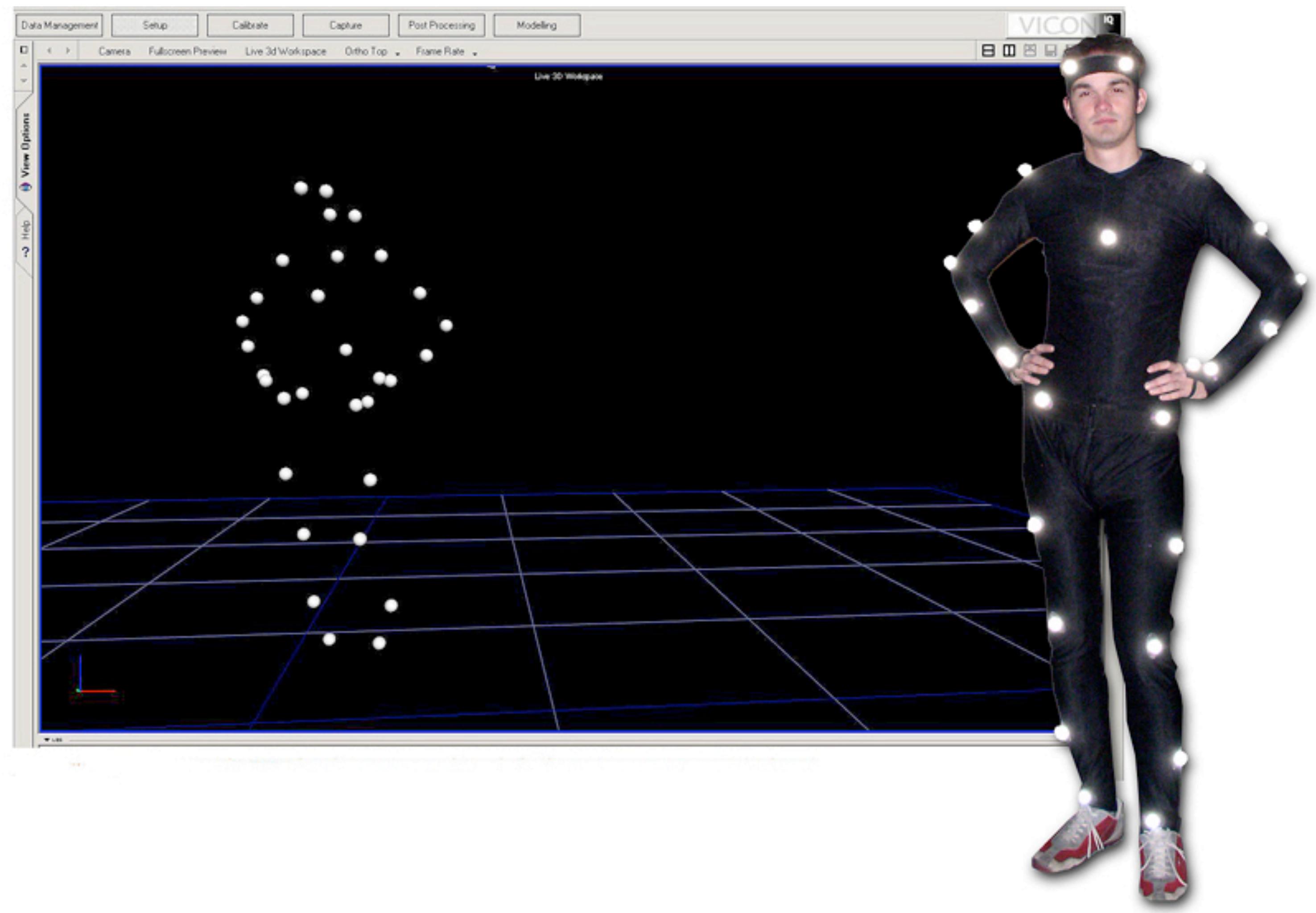
Outline

- **Application domains of camera networks**
  - Camera networks for motion capture
  - Camera network for video surveillance
- **Calibration of a camera network**
  - Mathematical modeling of the camera network calibration
  - Prior work
  - Description and motivation of the proposed solution
  - Example
  - How to distribute the algorithm
  - Simulations
  - Open issues

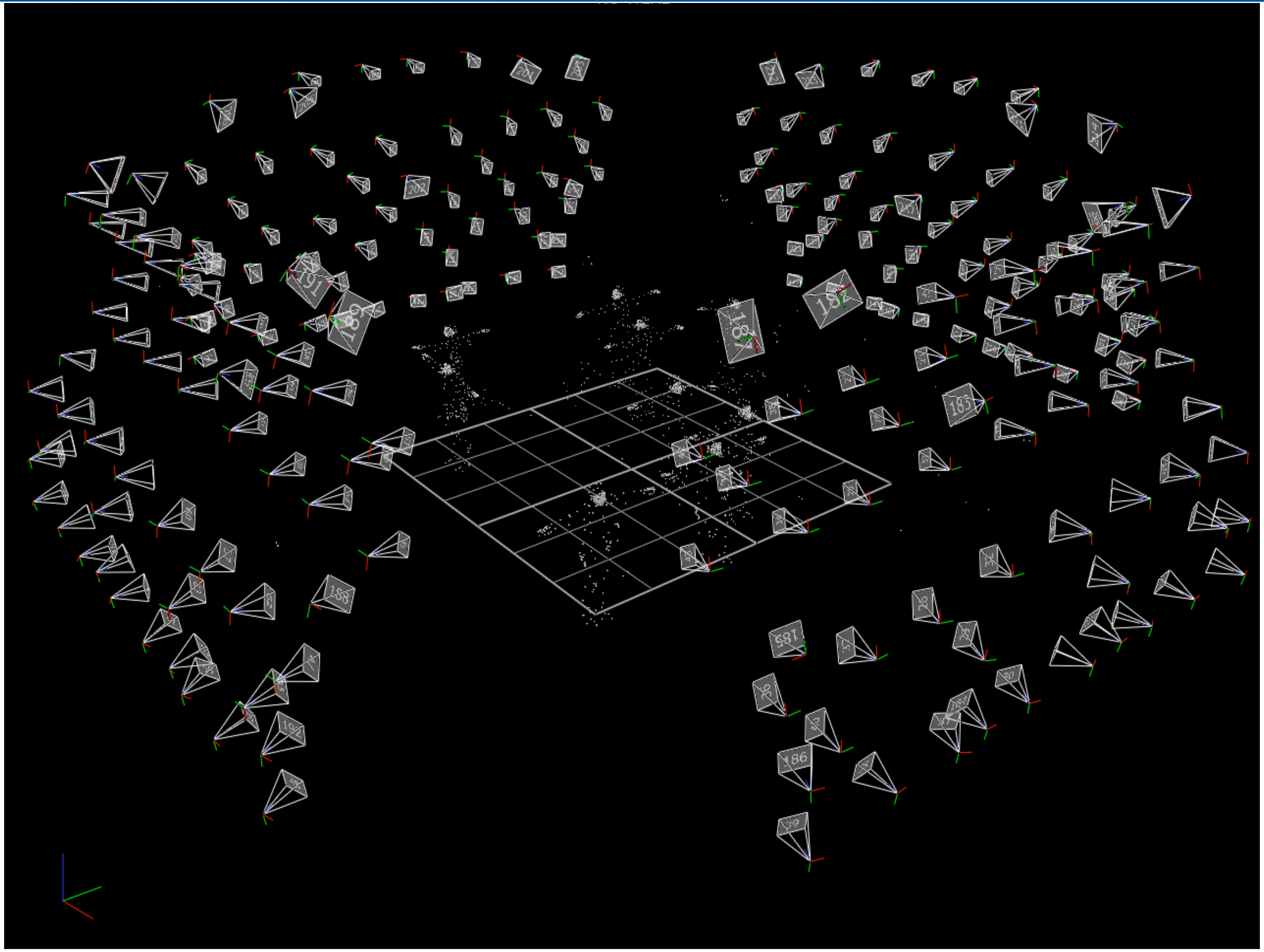
Camera networks for motion capture





- Reconstruction of the 3D motion from the tracking of **markers** done by a set of cameras which need to be calibrated
- High precision calibration is required

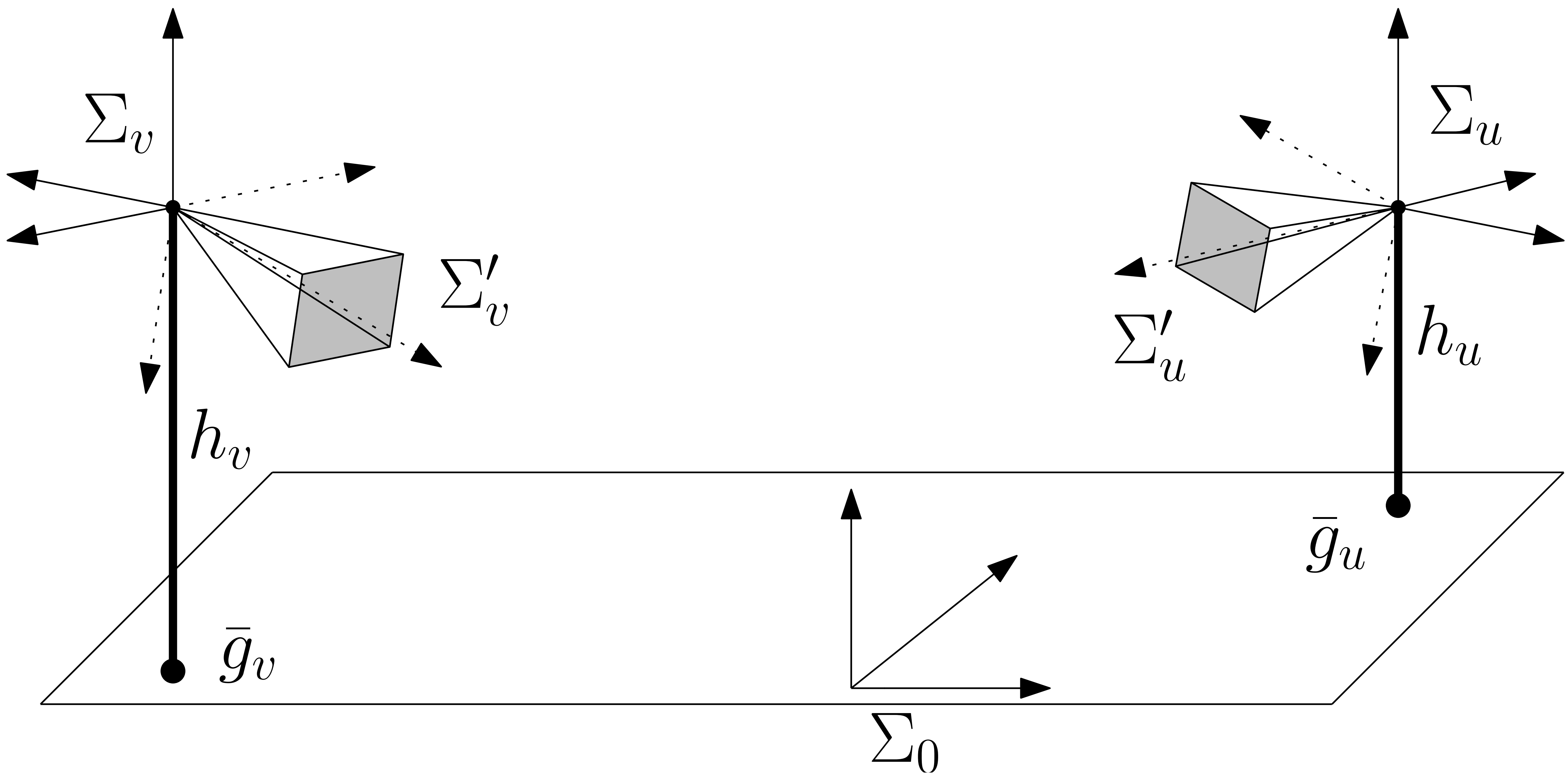
Camera networks for motion capture



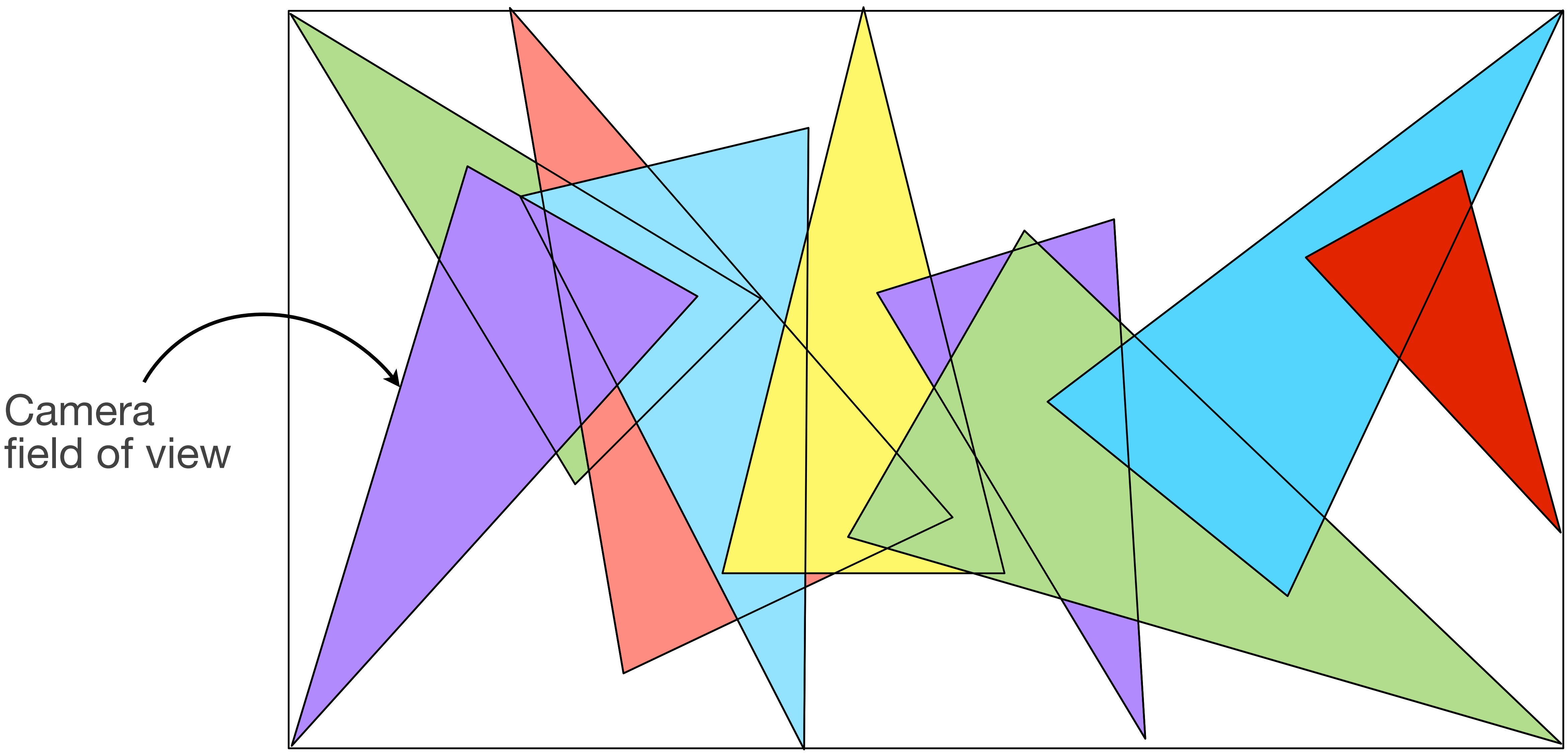
The required high precision and the coverage of a large space require a large number of cameras. To reduce the number of cameras it is convenient to use mobile cameras. Mobile cameras need real-time calibration.

Camera networks for video surveillance

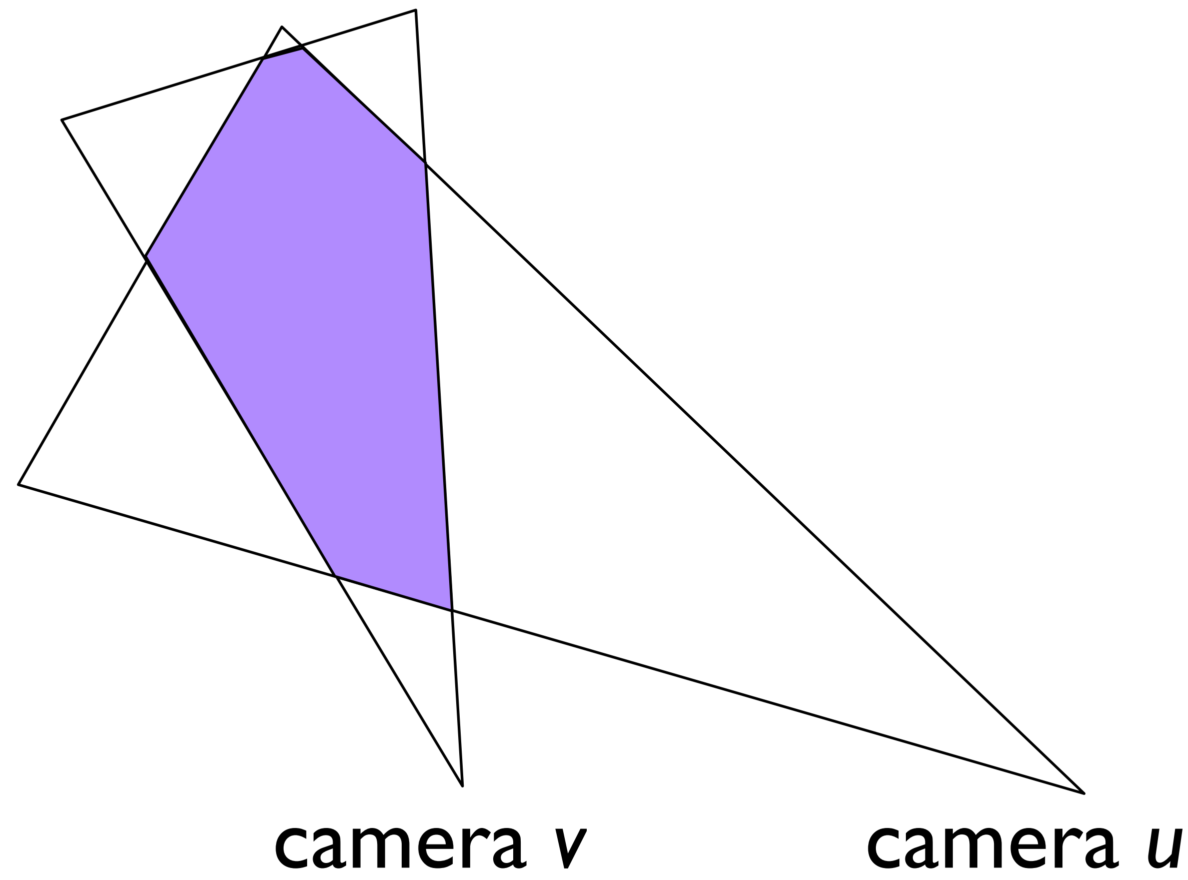
- A group of cameras used for perceiving some environment for surveillance purpose
- Low precision calibration is required



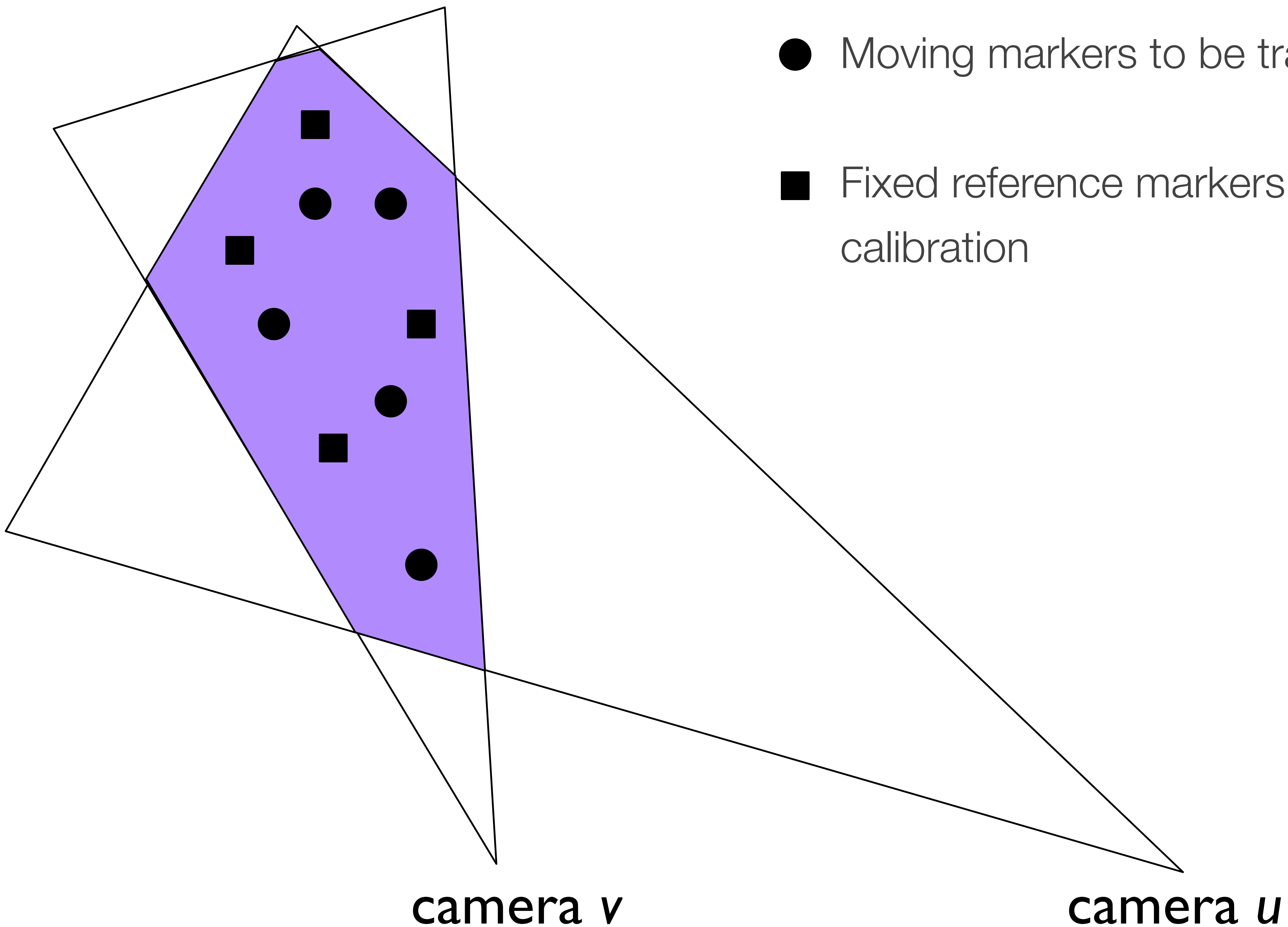
Σ_v
Σ'_v
h_v
$\bar{g}_v$
Σ_u
Σ'_u
h_u
$\bar{g}_u$
Σ_0



Simplified 2D modeling

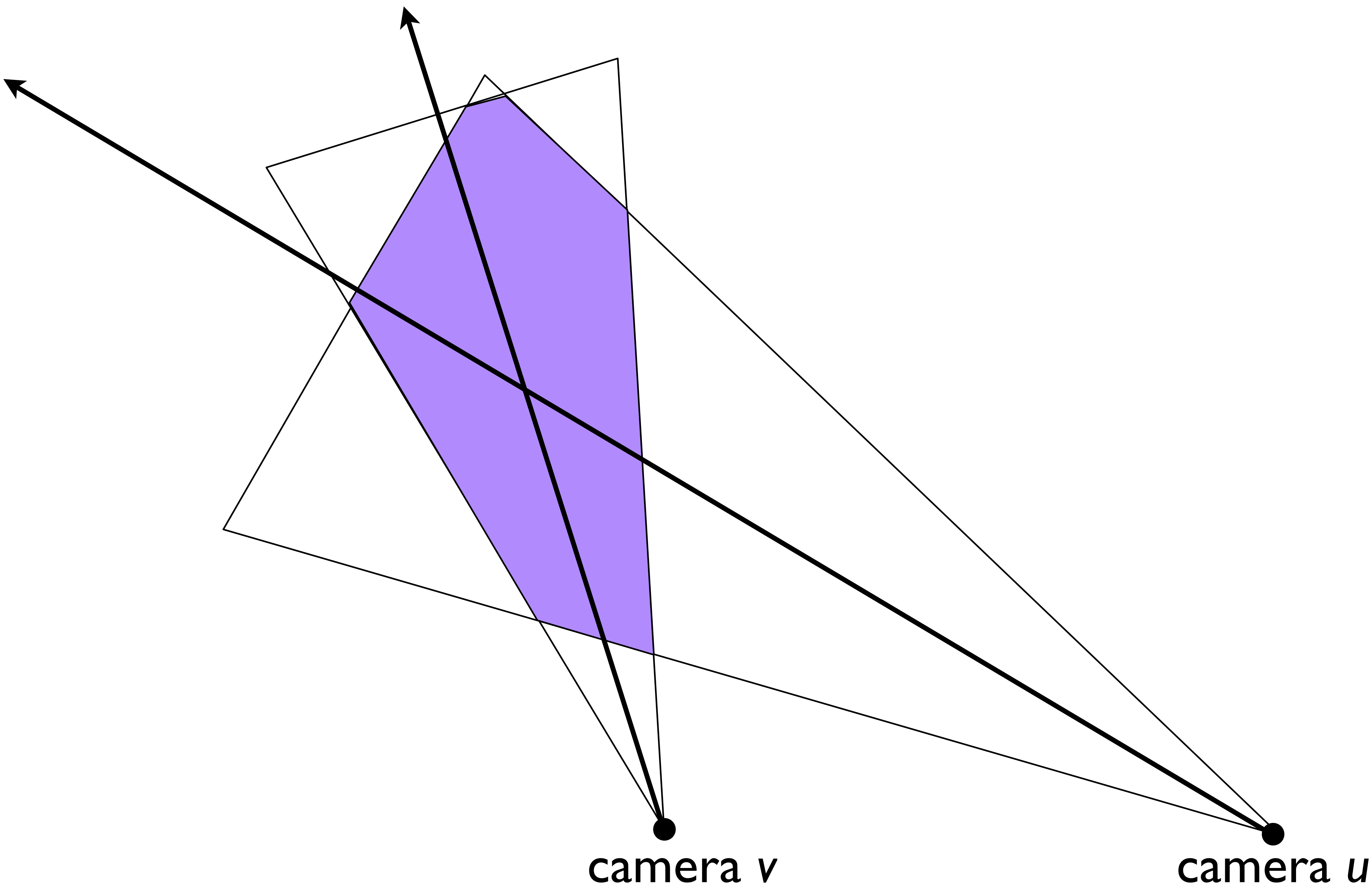

camera *v*
camera *u*

Calibration of camera networks



- Moving markers to be tracked
- Fixed reference markers for the calibration

Calibration of camera networks



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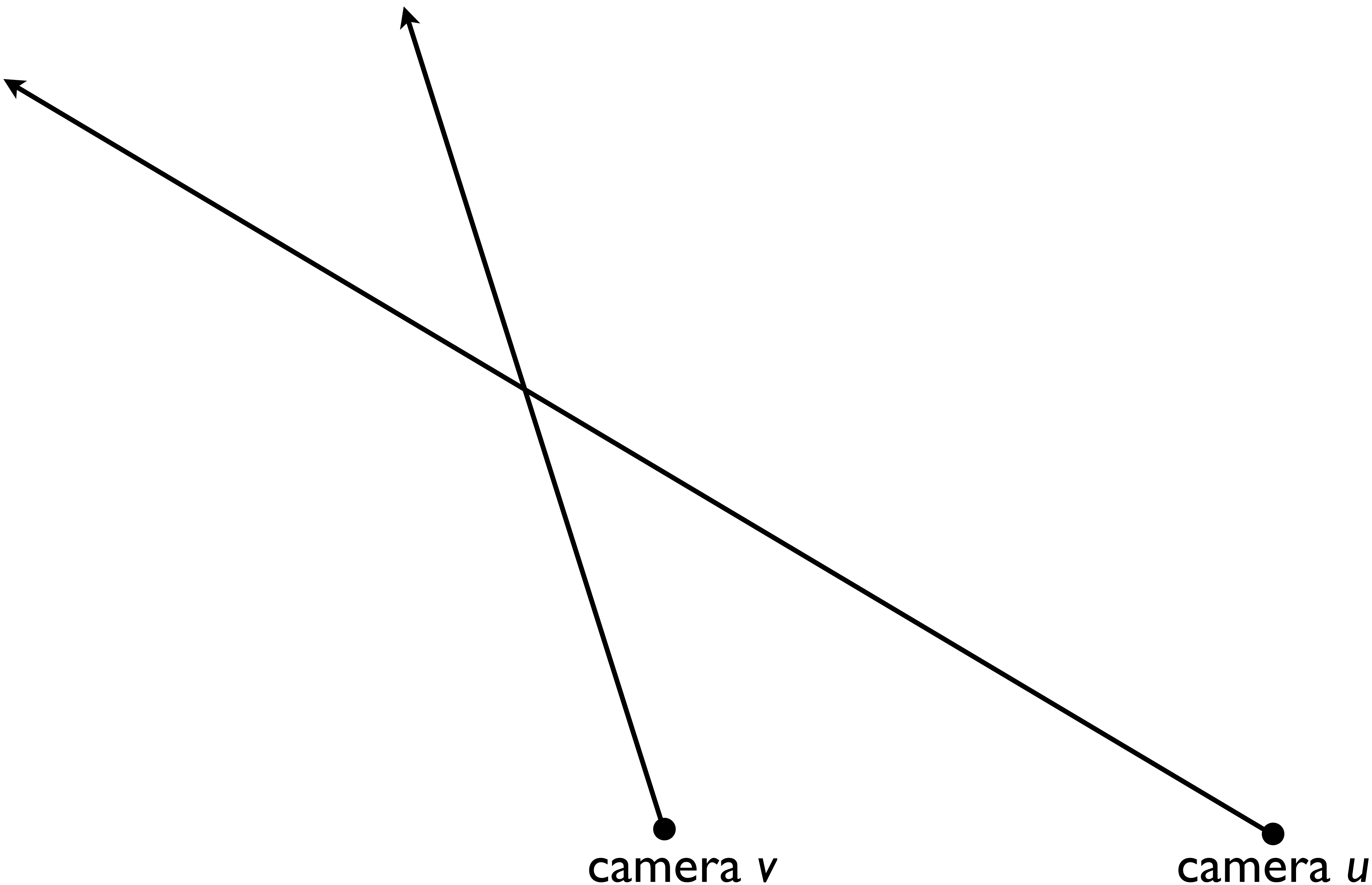
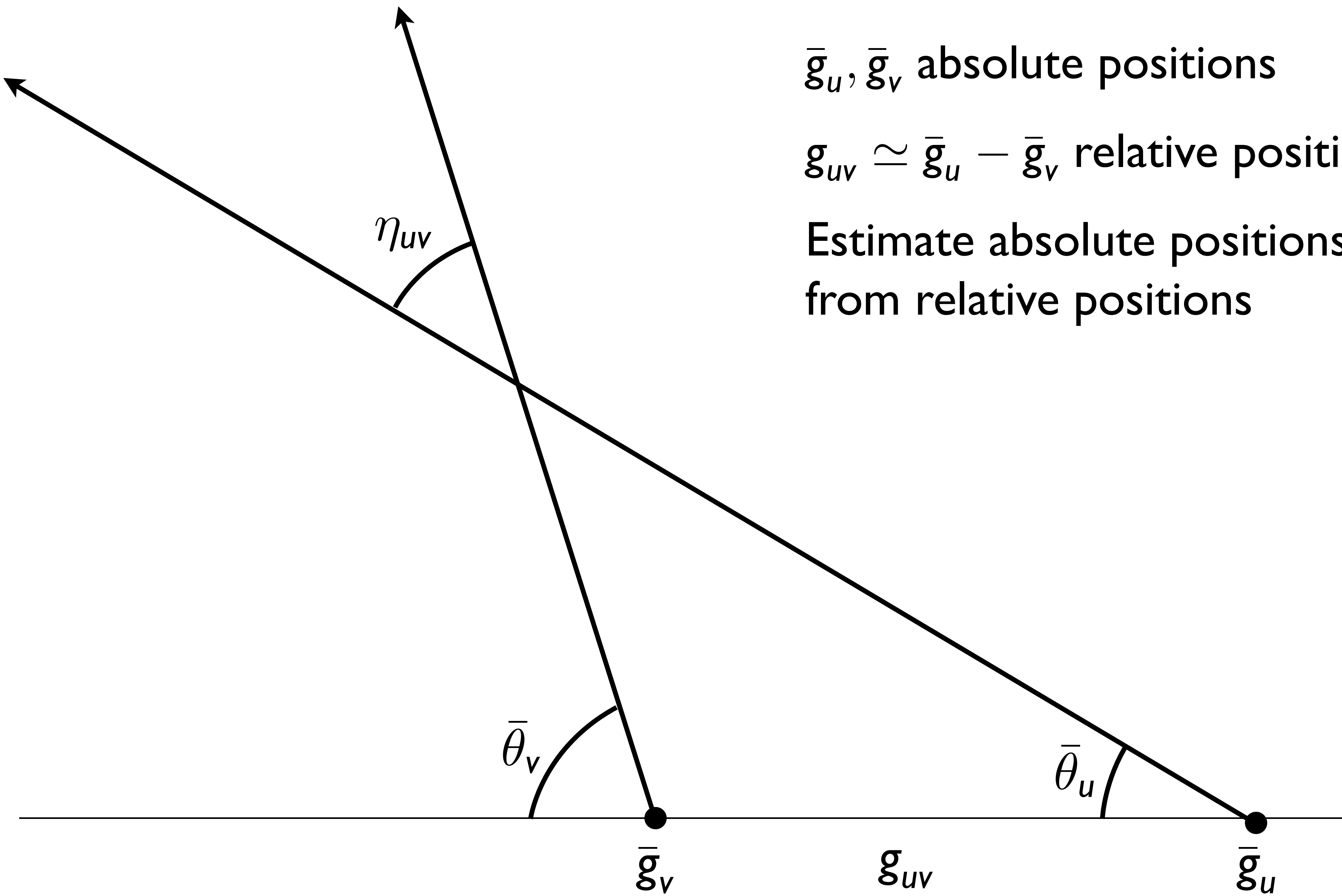
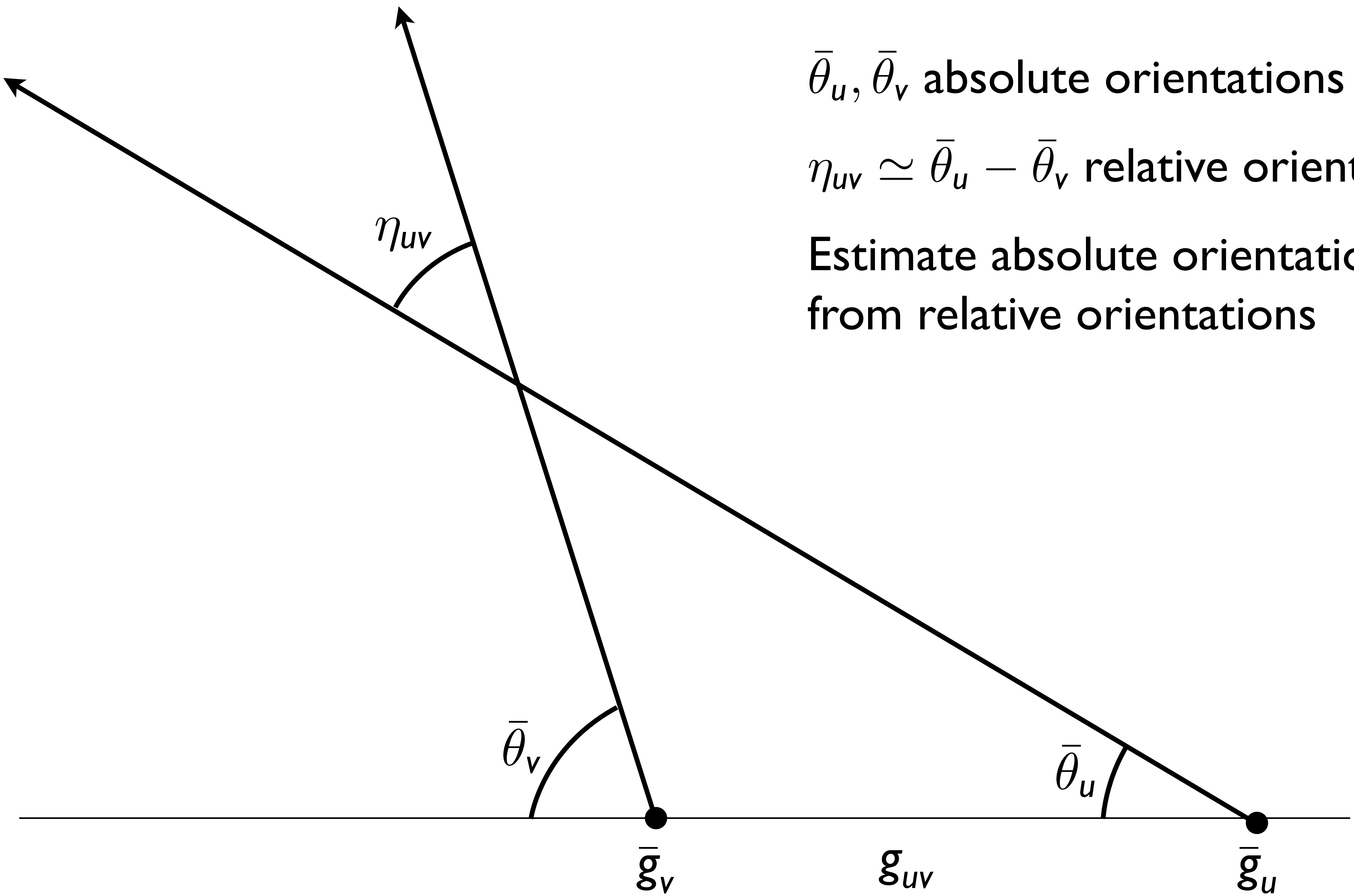




$\bar{g}_u, \bar{g}_v$ absolute positions

$g_{uv} \simeq \bar{g}_u - \bar{g}_v$ relative position

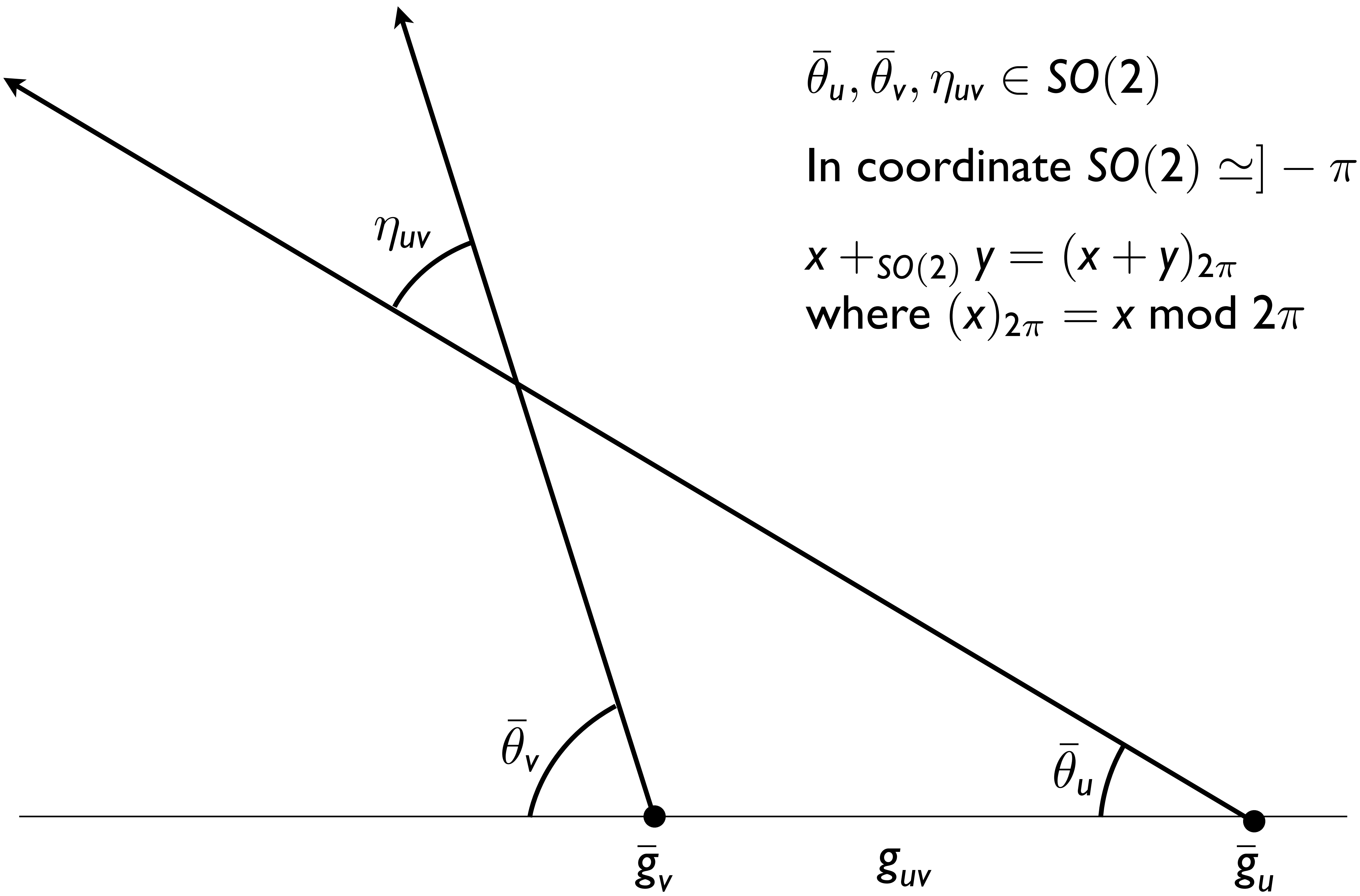
Estimate absolute positions from relative positions



$\bar{\theta}_u, \bar{\theta}_v$ absolute orientations

$\eta_{uv} \simeq \bar{\theta}_u - \bar{\theta}_v$ relative orientation

Estimate absolute orientations from relative orientations



$\bar{\theta}_u, \bar{\theta}_v, \eta_{uv} \in SO(2)$

In coordinate $SO(2) \simeq]-\pi, \pi]$

$x +_{SO(2)} y = (x+y)_{2\pi}$
where $(x)_{2\pi} = x \bmod 2\pi$

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A **directed graph** $\mathcal{G} = (V, \mathcal{E})$ is given, $\mathcal{E} \subseteq V \times V$. The matrix A is the incidence matrix of the graph $\mathcal{G}$

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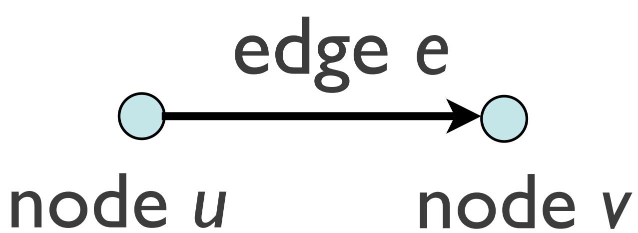
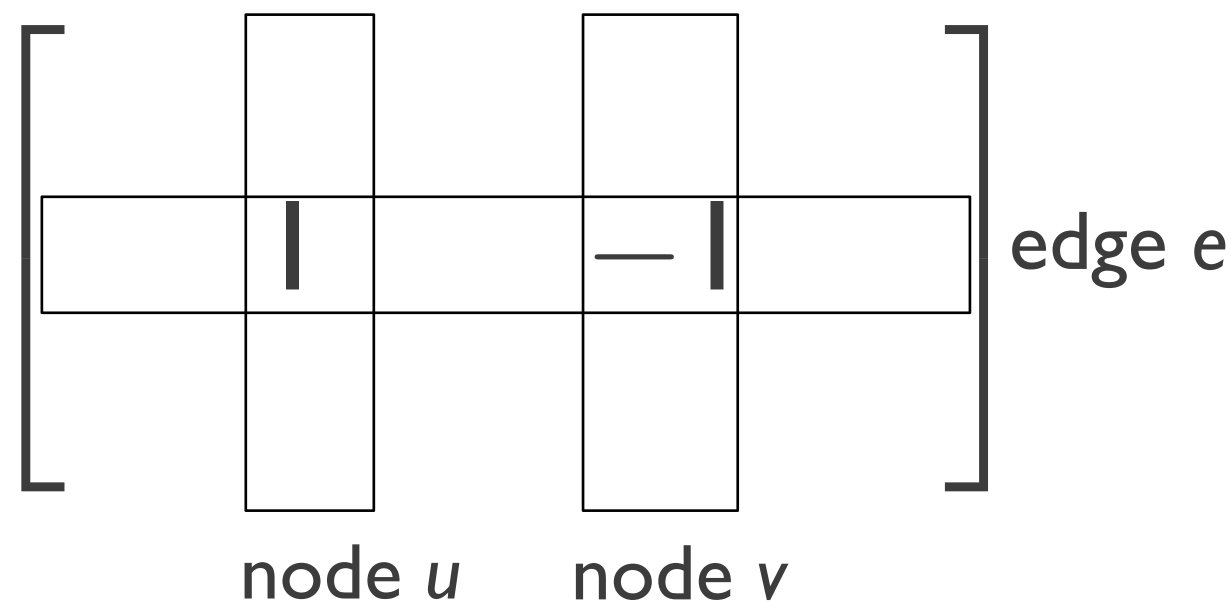
$$A \in \{-1, 0, 1\}^{\mathcal{E} \times V}$$



$A =$



$$(Ax)_e = x_u - x_v$$

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With each **node** $i \in V$ we associate the unknown *angle*

$$\bar{\theta}_i \in [-\pi, \pi) \qquad \bar{\boldsymbol{\theta}} = [\bar{\theta}_1, \ldots, \bar{\theta}_N]^T \in [-\pi, \pi)^V$$

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With each **edge** $\{i, j\} \in \mathcal{E}$ we associate the noisy relative measurement

$$\eta_{ij} = (\bar{\theta}_i - \bar{\theta}_j - \varepsilon_{ij})_{2\pi} \in [-\pi, \pi) \qquad \boldsymbol{\eta} = [\eta_{ij}] \in [-\pi, \pi)^M$$

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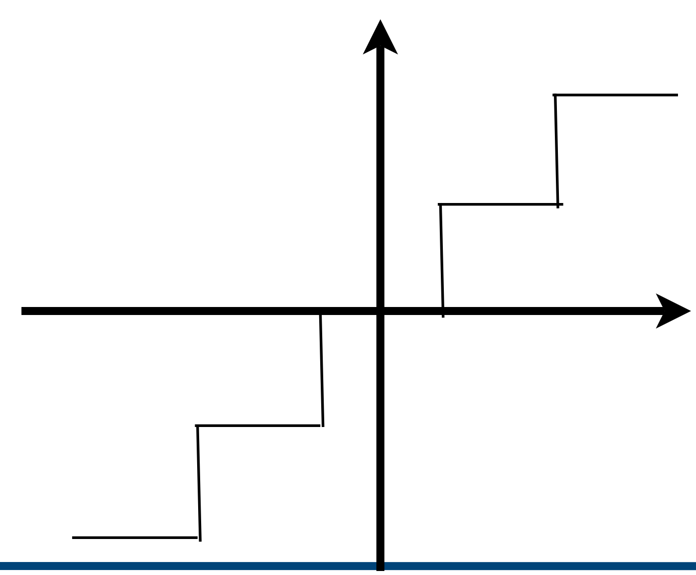
Notation:

$(x)_{2\pi} = x \bmod 2\pi$

$q_{2\pi}(x) = \lfloor \frac{x+\pi}{2\pi} \rfloor$

$$x = (x)_{2\pi} + 2\pi q_{2\pi}(x)$$



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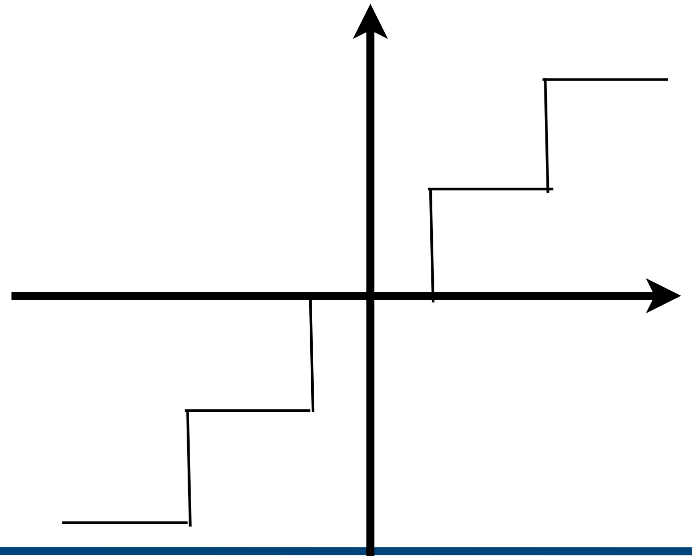

$$\boldsymbol{\eta} = (A\bar{\boldsymbol{\theta}} + \boldsymbol{\varepsilon})_{2\pi}$$

Modeling of the angular calibration

Assumption: With no loss of generality we can assume that $\bar{\theta}_1 = 0$.

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Possible solution: minimize the cost function on $[-\pi, \pi)^N$

$$V(\boldsymbol{\theta}) := ||(A\boldsymbol{\theta} - \boldsymbol{\eta})_{2\pi}||^2 = \min_{K \in \mathbb{Z}^{\mathcal{E}}} ||A\boldsymbol{\theta} - \boldsymbol{\eta} - 2\pi K||^2$$

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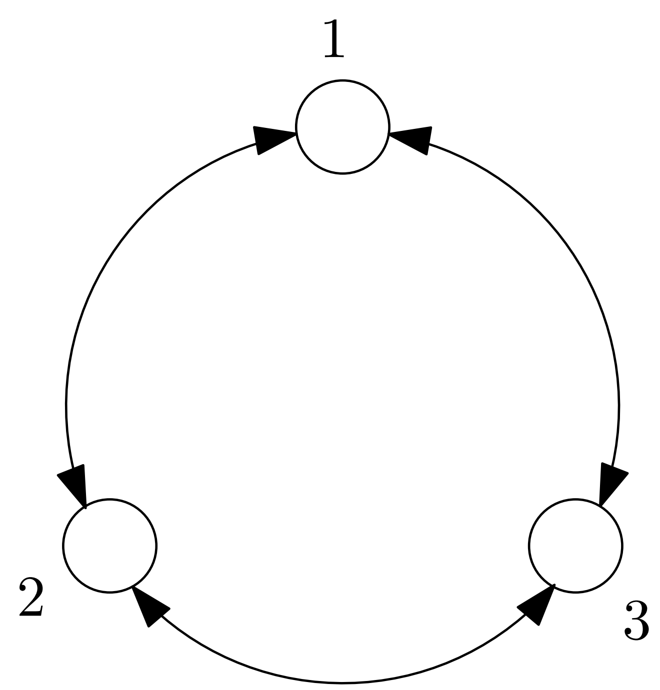
Fact: $V(\boldsymbol{\theta})$ has several local minima due to the geometry of the circle

Modeling of the angular calibration

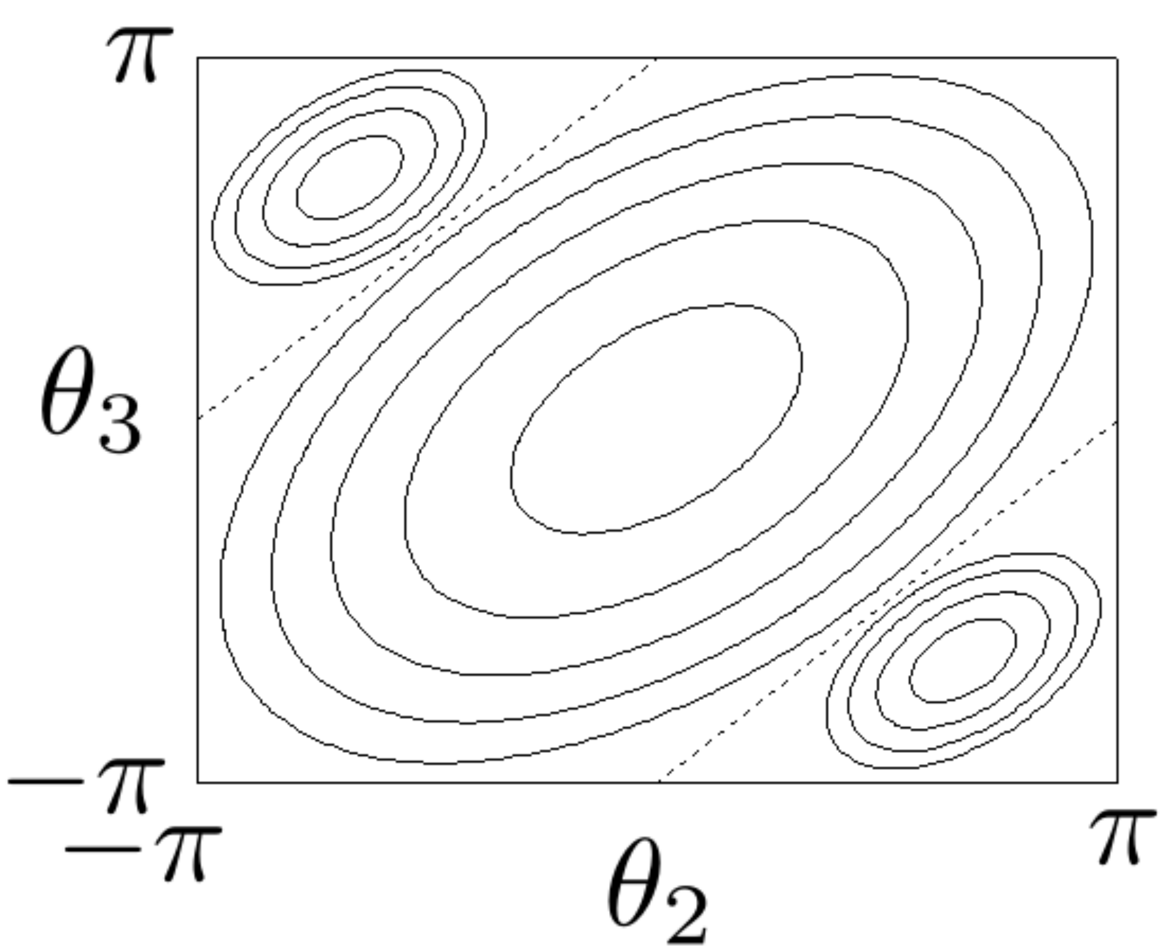
Example: 3 cameras with $\bar{\theta}_1 = \bar{\theta}_2 = \bar{\theta}_3 = 0$ and a circular graph $\mathcal{G}$



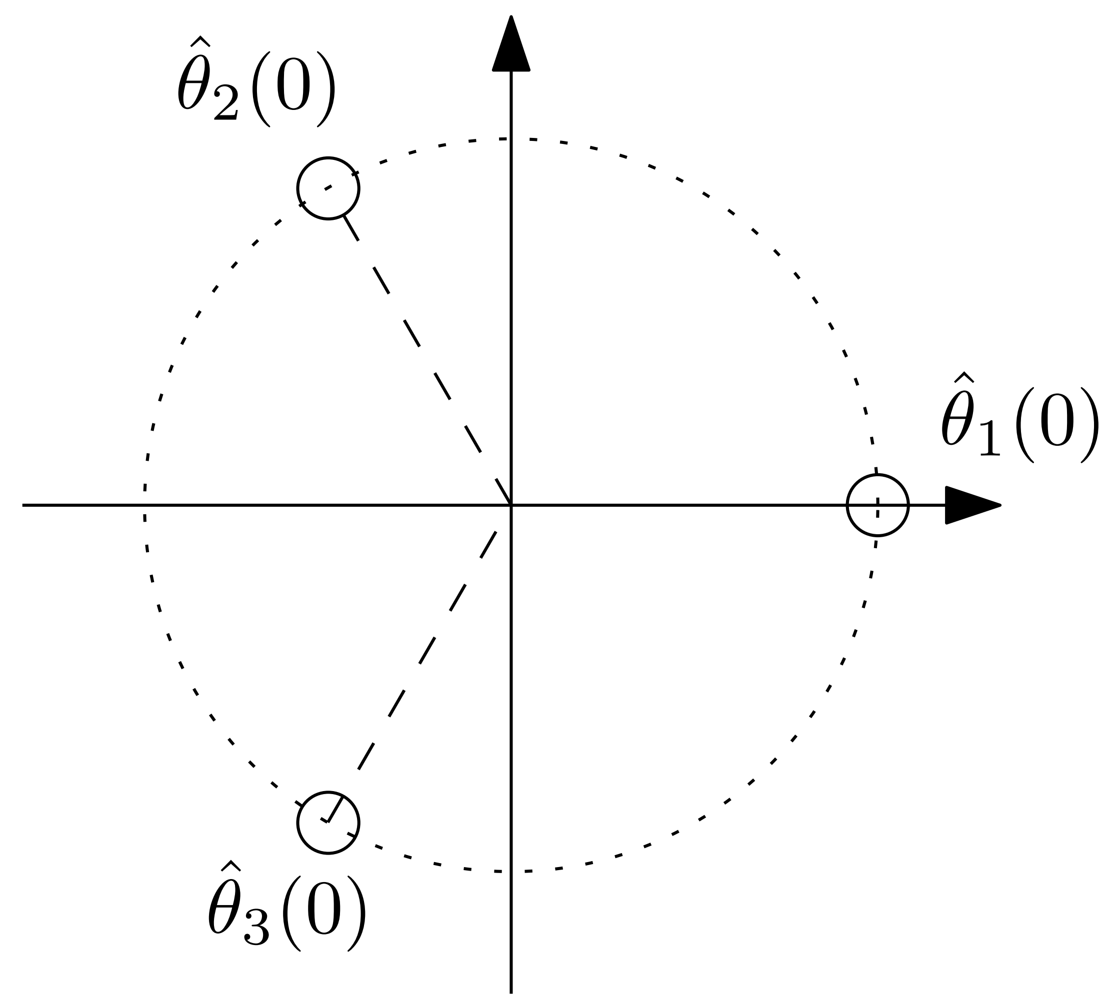
Noiseless case so that $\eta_{12} = \eta_{23} = \eta_{31} = 0$

Modeling of the angular calibration

Assumption: $\bar{\theta}_1 = 0$ so that $V(\boldsymbol{\theta}) = V(\theta_2, \theta_3)$ for $\theta_2, \theta_3 \in (-\pi, \pi]$.



Contour lines of $V(\boldsymbol{\theta}) = V(\theta_2, \theta_3)$



Local minimum

Prior work

Estimation of the *position* of the agents with a partial knowledge of their *relative position*.

Setting: $\theta \in \mathbb{R}^V$

$$A\eta = A\theta + \varepsilon$$

The proposed estimation $\hat{\theta}$ is the minimizer of the quadratic cost

$$V(\theta) := \|A\theta - \eta\|^2$$

- A distributed consensus type estimator is proposed.
- The performance is related to the effective resistance of the graph.

P. Barooah, J. Hespahna, "Estimation on Graphs from relative Measurements: Distributed Algorithms and Fundamental Limits", IEEE Control Systems Magazine, vol. 27, 2007.

S. Bolognani, S. Del Favero, L. Schenato, D. Varagnolo. "Consensus-based distributed sensor calibration and least-square parameter identification in WSNs", International Journal of Robust and Nonlinear Control, vol. 20, 2010.

Prior work

Consensus over lie groups and manifolds:

A. Sarlette, S. Bonnabel and R. Sepulchre, S Bonnabel
R. Tron, B. Afsari, R. Vidal, A Terzis
Y. Igarashi, T. Hatanaka, M. Fujita, M.W. Spong

Distributed algorithms for angular calibration:

G. Piovan, I. Shames, B. Fidan, F. Bullo, and B. D. O. Anderson. "On Frame and Orientation Localization for Relative Sensing Networks". Automatica, February 2011.

Optimization over manifolds:

R. Sepulchre, "Optimization over matrix manifolds", Princeton University Press, 2008.

Proposed solution

Proposed solution: Observe that

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}\in\mathbb{R}^V\in\mathbb{Z}^{\mathcal{E}}}{\operatorname{argmin}} \|(A\boldsymbol{\theta} - \boldsymbol{\eta})_{2\pi}\|^2$$

and

$$(\hat{\boldsymbol{\theta}}, \hat{K}) := \underset{\boldsymbol{\theta}\in\mathbb{R}^V, K\in\mathbb{Z}^{\mathcal{E}}}{\operatorname{argmin}} \|A\boldsymbol{\theta} - \boldsymbol{\eta} - 2\pi K\|^2$$

give the same answer $\hat{\boldsymbol{\theta}}$.

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From $\hat{K}$ it is easy to determine $\hat{\boldsymbol{\theta}}$ using standard quadratic optimization formulas

$$\hat{\boldsymbol{\theta}} = (A^T A)^\sharp A^T (\boldsymbol{\eta} + 2\pi\hat{K})$$

where $(A^T A)^\sharp$ is a pseudo inverse of $A^T A$.

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$$\hat{\boldsymbol{\theta}} = (A^T A)^{\sharp} A^T (\boldsymbol{\eta} + 2\pi \hat{K})$$

where $(A^T A)^{\sharp}$ is a pseudo inverse of $A^T A$.

Notice that there are standard distributed algorithms for determining $\hat{\boldsymbol{\theta}}$.

Proposed solution

For fixed K let

$$\hat{\boldsymbol{\theta}}(K) := \underset{\boldsymbol{\theta}\in\mathbb{R}^V}{\operatorname{argmin}} \|(A\boldsymbol{\theta} - \boldsymbol{\eta} - 2\pi K\|^2 = (A^T A)^\sharp A^T(\boldsymbol{\eta} + 2\pi K)$$

and let

$$f(K) := \|(A\hat{\boldsymbol{\theta}} - \boldsymbol{\eta} - 2\pi K\|^2$$

so that

$$\hat{K} = \underset{K\in\mathbb{Z}^{\mathcal{E}}}{\operatorname{argmin}}\, f(K)$$

It can be shown that

$$f(K) = (\boldsymbol{\eta} - 2\pi K)^T (I - A(A^T A)^\sharp A^T)(\boldsymbol{\eta} - 2\pi K)$$

It is **difficult** to obtain

$$\hat{K} = \underset{K\in\mathbb{Z}^{\mathcal{E}}}{\operatorname{argmin}}\, f(K)$$

Proposed solution

With any closed path (cycle) γ on the graph $\mathcal{G}$ we can associate a column $R_\gamma \in \mathbb{Z}^{\mathcal{E}}$ such that

$$\sum_{\{i,j\}\in\gamma} \eta_{ij} = R_\gamma^T \boldsymbol{\eta}$$

Notice that

$$R_\gamma^T A = 0$$

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It can be shown that

$$f(K) = \|(R^T R)^{-1/2} R^T (\boldsymbol{\eta} - 2\pi K)\|^2$$

Proposed solution

From this we can argue that

$$\frac{1}{\sigma_{\max}(R)}\|R^T(\boldsymbol{\eta} - 2\pi K)\|^2 \leq f(K) \leq \frac{1}{\sigma_{\min}(R)}\|R^T(\boldsymbol{\eta} - 2\pi K)\|^2$$

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If $\sigma_{\min}(R) \simeq \sigma_{\max}(R)$ then

$$\hat{K} \simeq \underset{K \in \mathbb{Z}^{\mathcal{E}}}{\operatorname{argmin}} \|R^T(\boldsymbol{\eta} - 2\pi K)\|^2$$

Proposed solution

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$$\frac{1}{\sigma_{\max}(R)}\|R^T(\boldsymbol{\eta}-2\pi K)\|^2 \leq f(K) \leq \frac{1}{\sigma_{\min}(R)}\|R^T(\boldsymbol{\eta}-2\pi K)\|^2$$

If $\sigma_{\min}(R) \simeq \sigma_{\max}(R)$ then

$$\hat{K} \simeq \operatorname*{argmin}_{K\in\mathbb{Z}^{\mathcal{E}}} \|R^T(\boldsymbol{\eta}-2\pi K)\|^2$$

This can be found easily

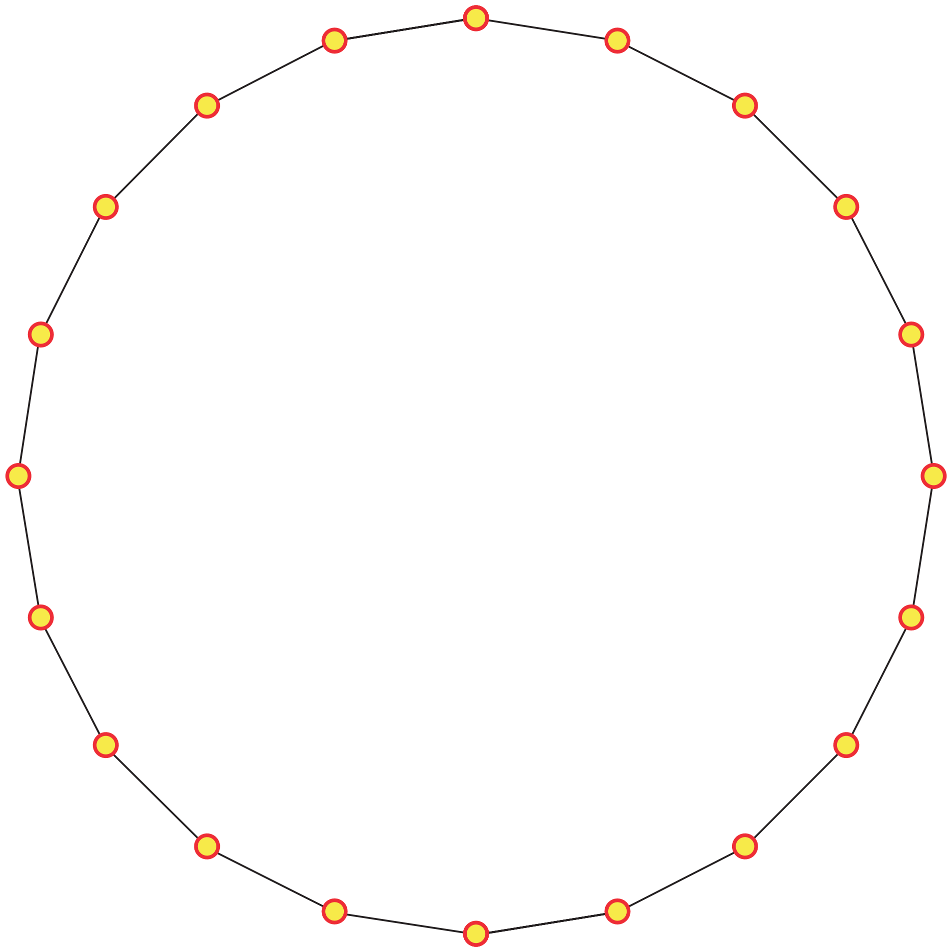
$$\operatorname*{argmin}_{K\in\mathbb{Z}^{\mathcal{E}}} \|R^T(\boldsymbol{\eta}-2\pi K)\|^2 = Xq_{2\pi}\left(R^T\boldsymbol{\eta}\right)$$

where X is a matrix with entries in $\mathbb{Z}$ such that $R^TX = I$.

Example

Consider the cycle graph. For this the incidence matrix is

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$



and

$$R = [1\ 1\ 1\ \cdots\ 1\ 1]^T$$

In this case, since $\sigma_{\min}(R) = \sigma_{\max}(R) = \sqrt{N}$, then

$$\hat{K} = \underset{K \in \mathbb{Z}^{\mathcal{E}}}{\operatorname{argmin}} \| \sum_e \eta_e - 2\pi \sum_e K_e \|^2 = X q_{2\pi}\left(\sum_e \eta_e \right)$$

where X is a left inverse of $R^T = [1\ 1\ 1\ \cdots\ 1\ 1]$.

Questions to be answered

- Is it possible to find a distributed algorithm for determining this estimate $\hat{K}$?
  - Yes, but it depends on the choice of the "basis" of cycles we make.
- Is it possible to make an error analysis of the algorithm based on a given statistical description of the noise?
  - Only partially so far. We could not obtain a good statistical description of the error in the estimation $\hat{K}$ of $\bar{K}$. We know the behavior of

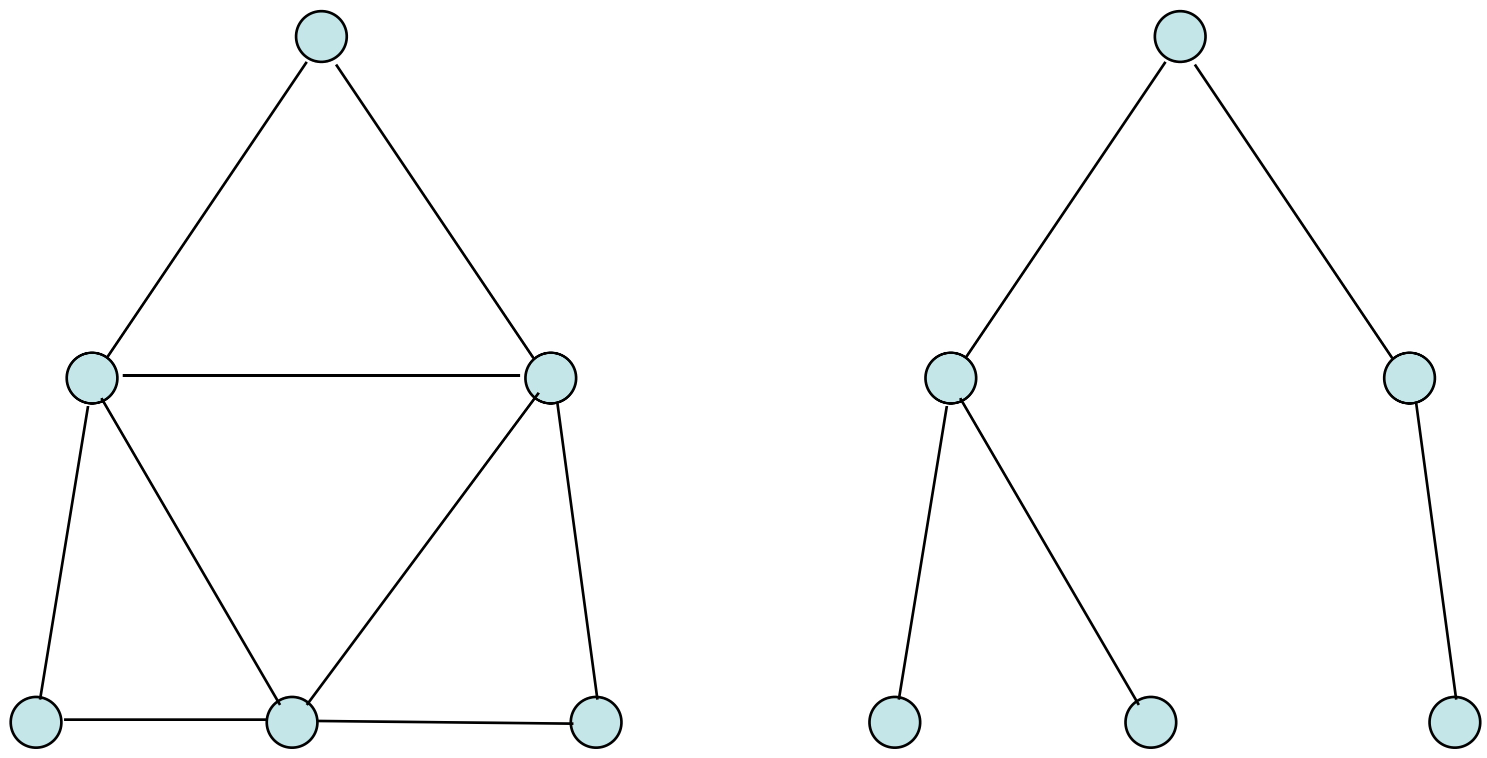
$$E(\|\hat{\theta} - \hat{\theta}\| \;\mid\; \hat{K} = \bar{K})$$

but not of

$$E(\|\hat{\theta} - \hat{\theta}\|) = \sum_{\tilde{K} \in \mathbb{Z}^{\mathcal{E}}} E(\|\hat{\theta} - \hat{\theta}\| \;\mid\; \hat{K} = \bar{K} + \tilde{K}) P(\hat{K} = \bar{K} + \tilde{K})$$

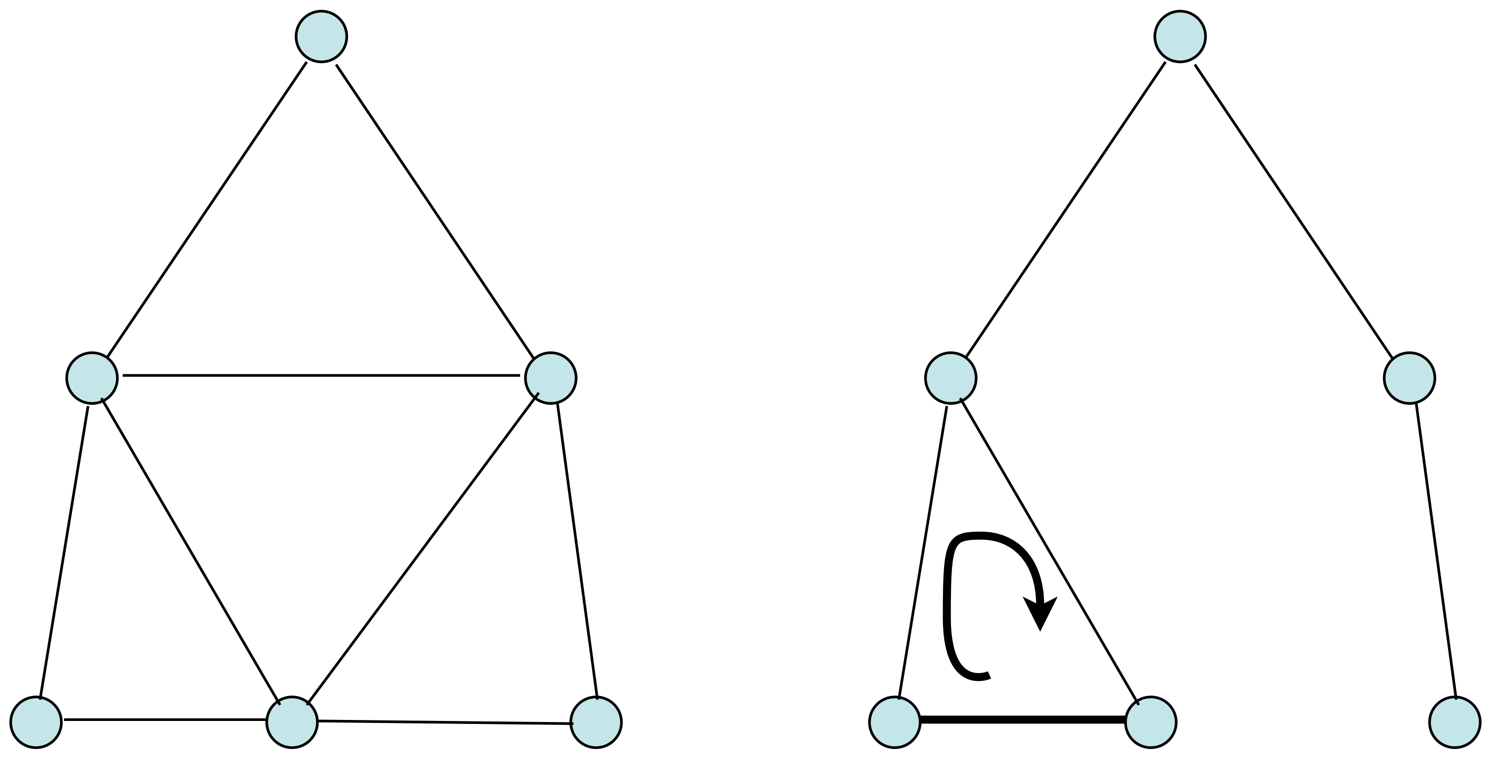
How to distribute the algorithm

Families of cycles for which the algorithm can be implemented in a **distributed** way start from spanning tree of the graph.



How to distribute the algorithm

Fundamental cycles: easier to distribute but with bigger reconstruction error.



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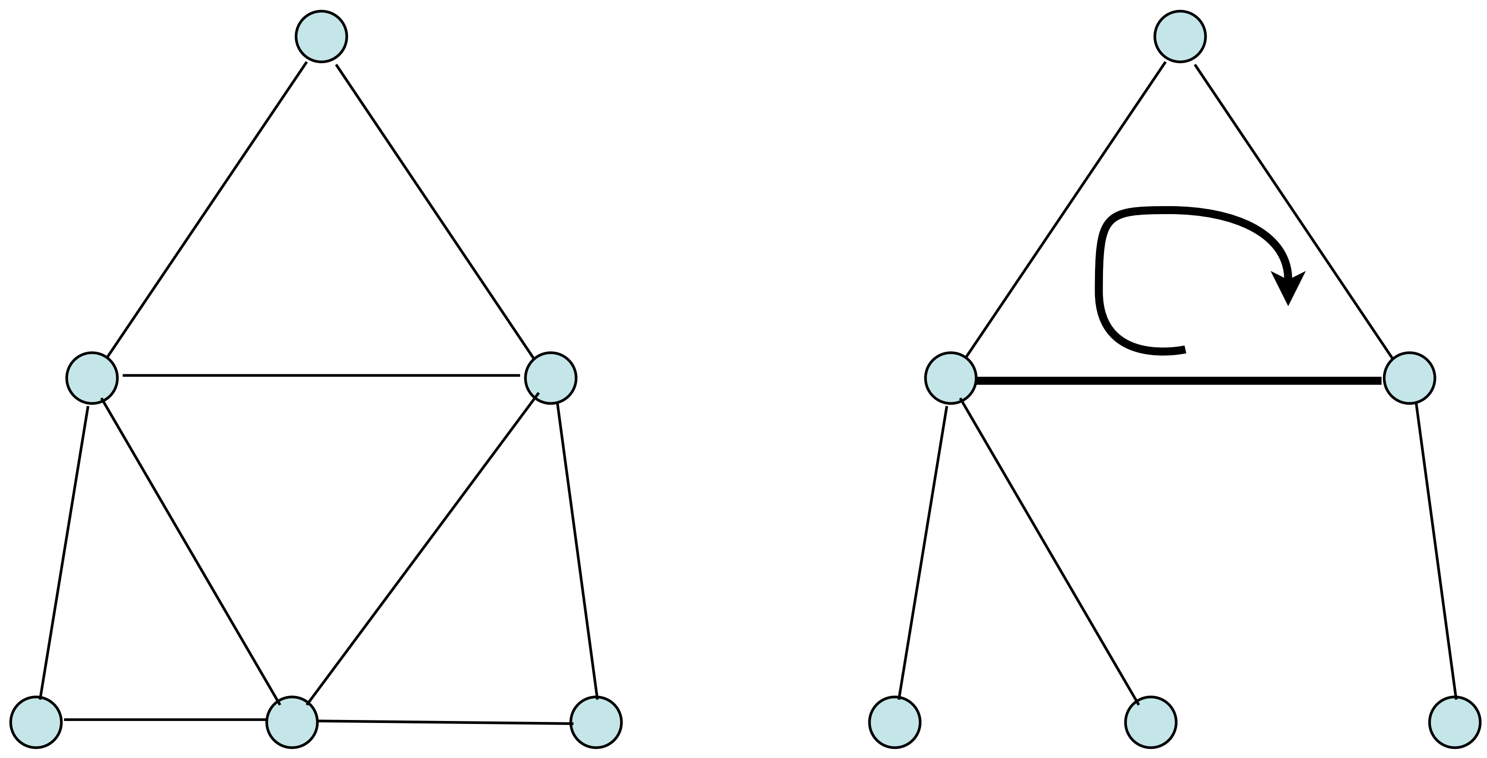

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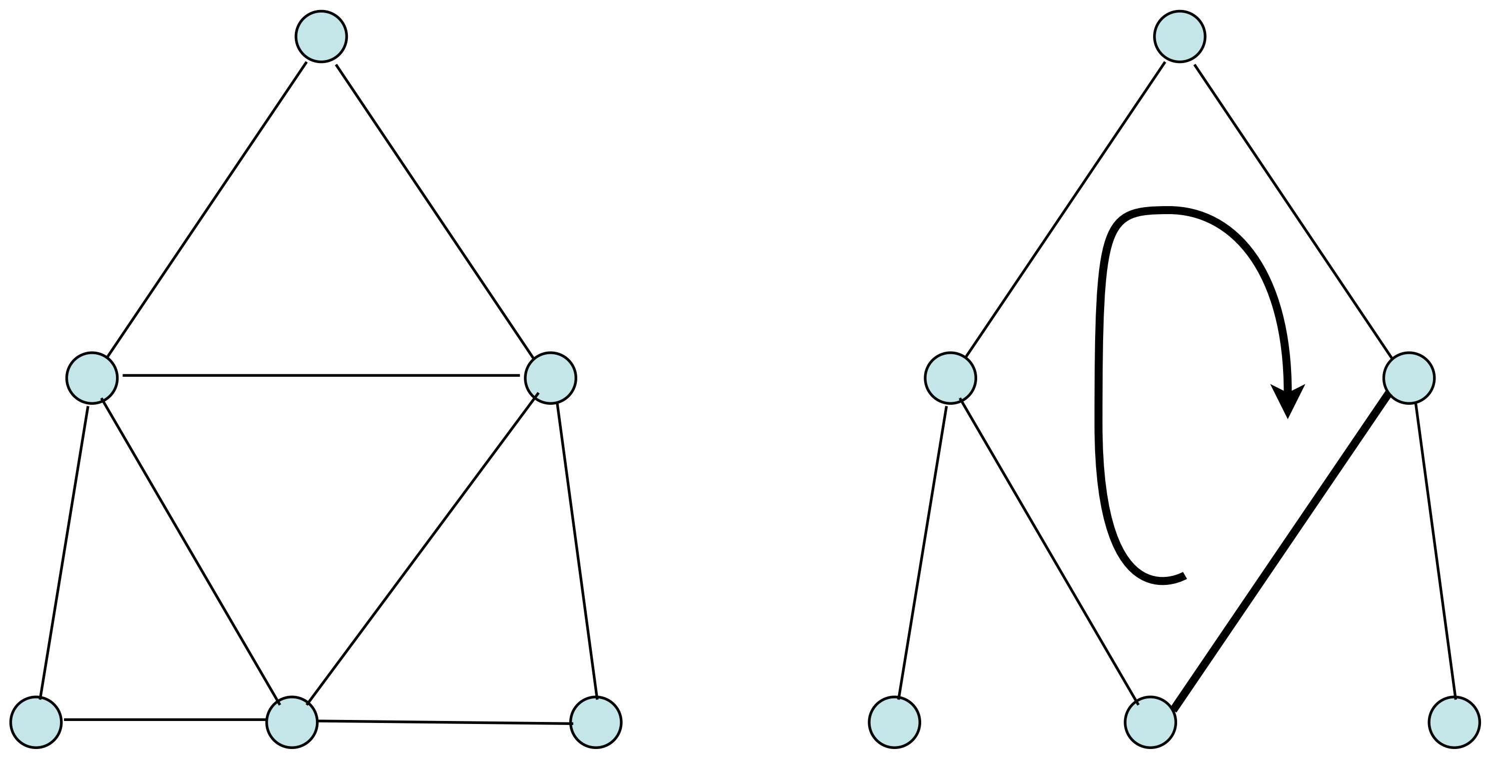

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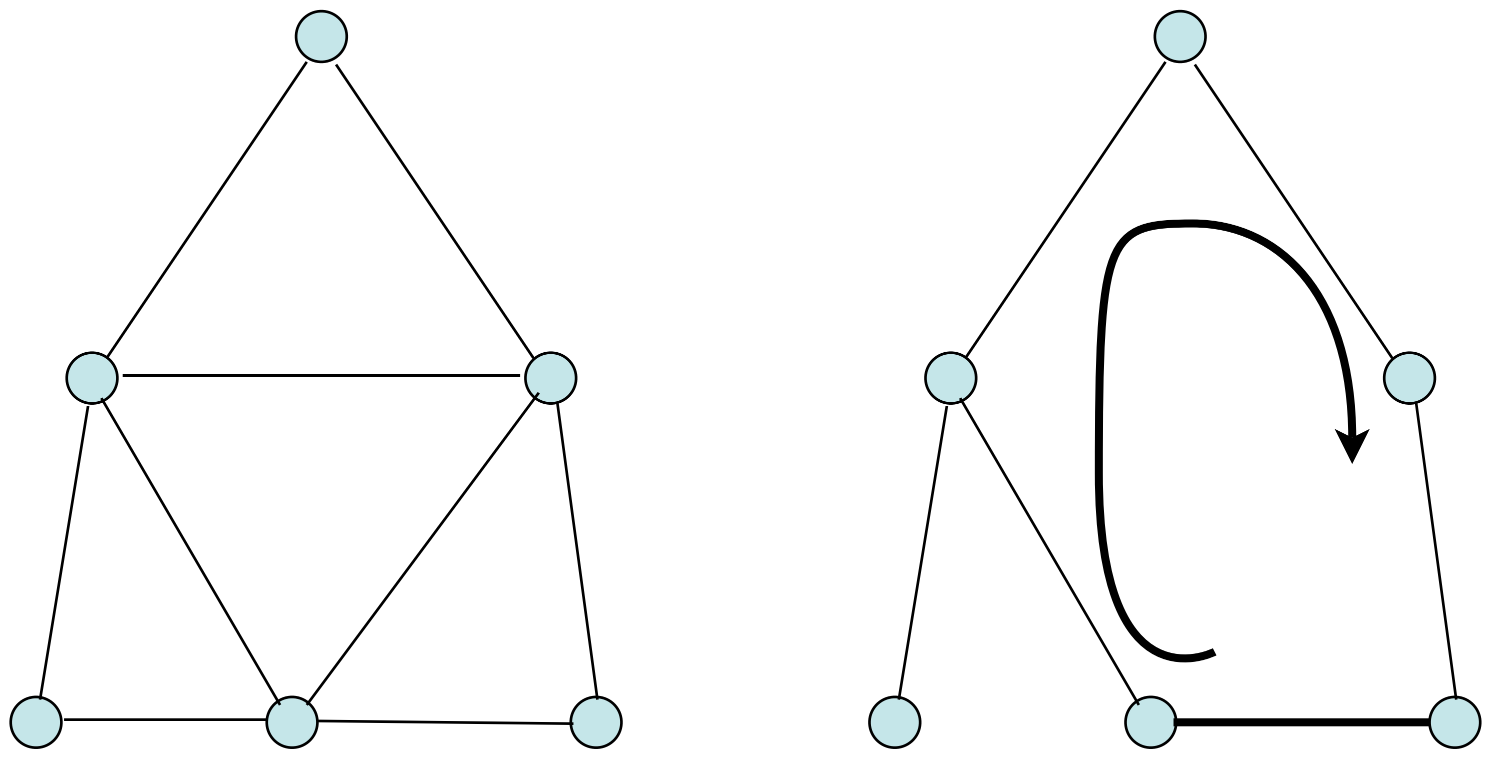

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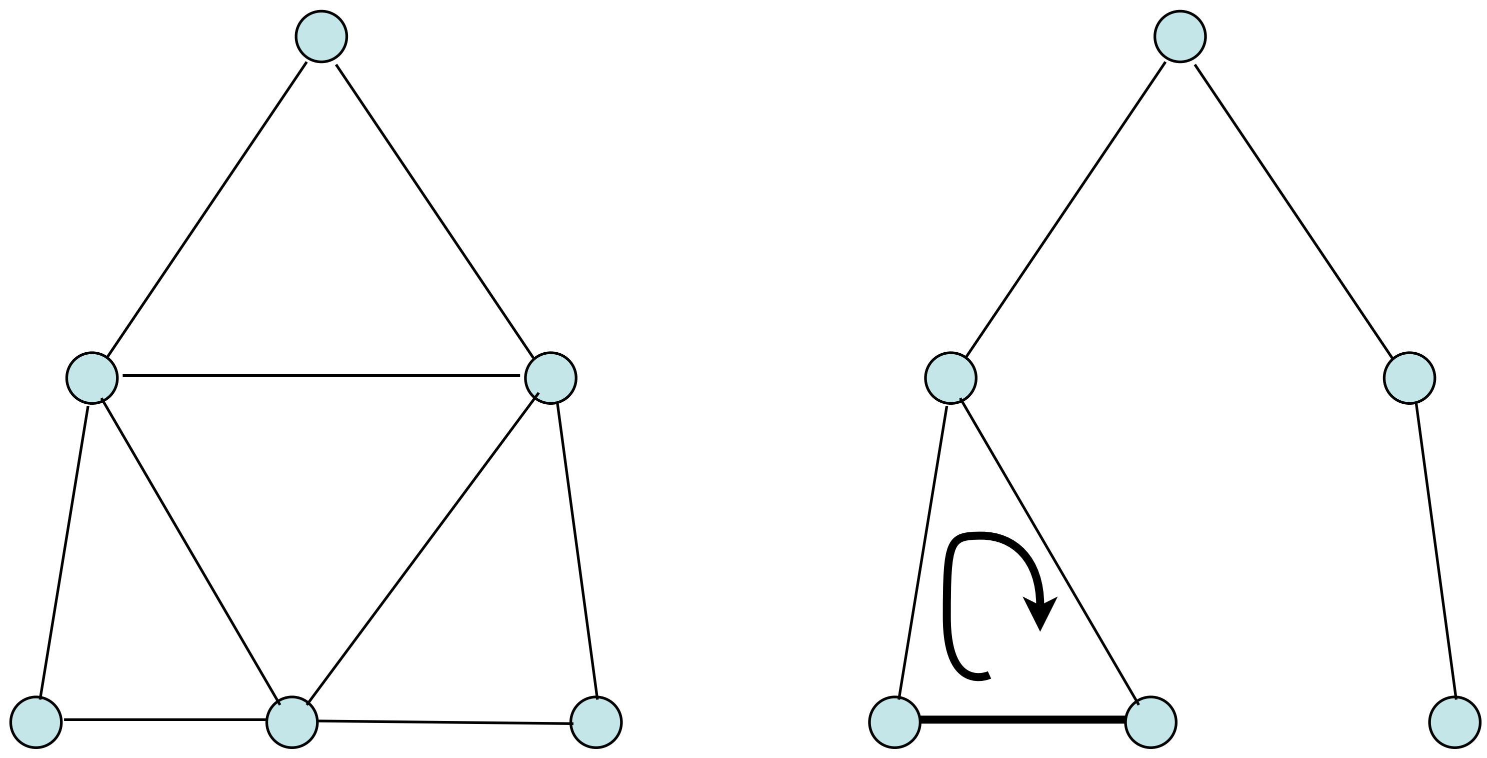

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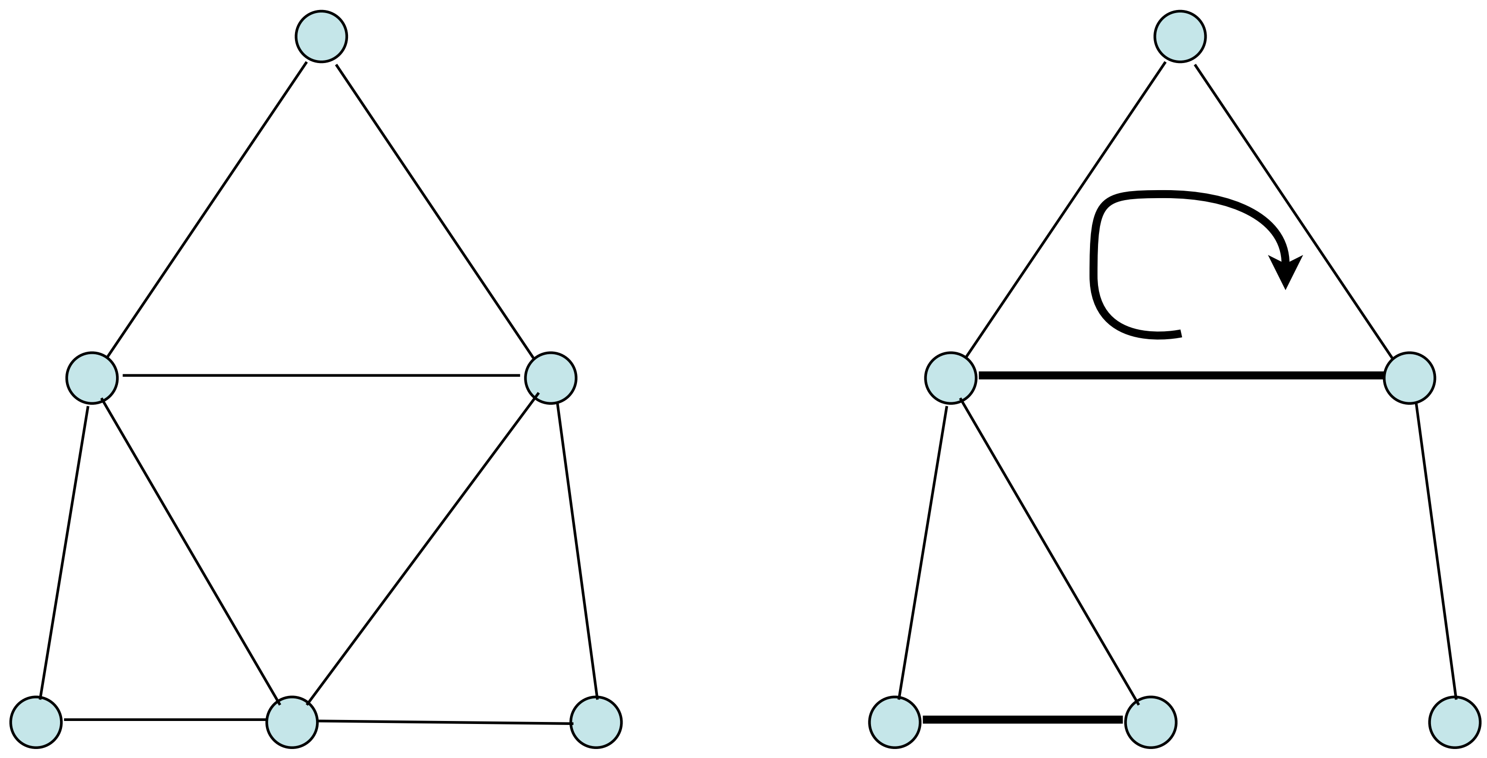

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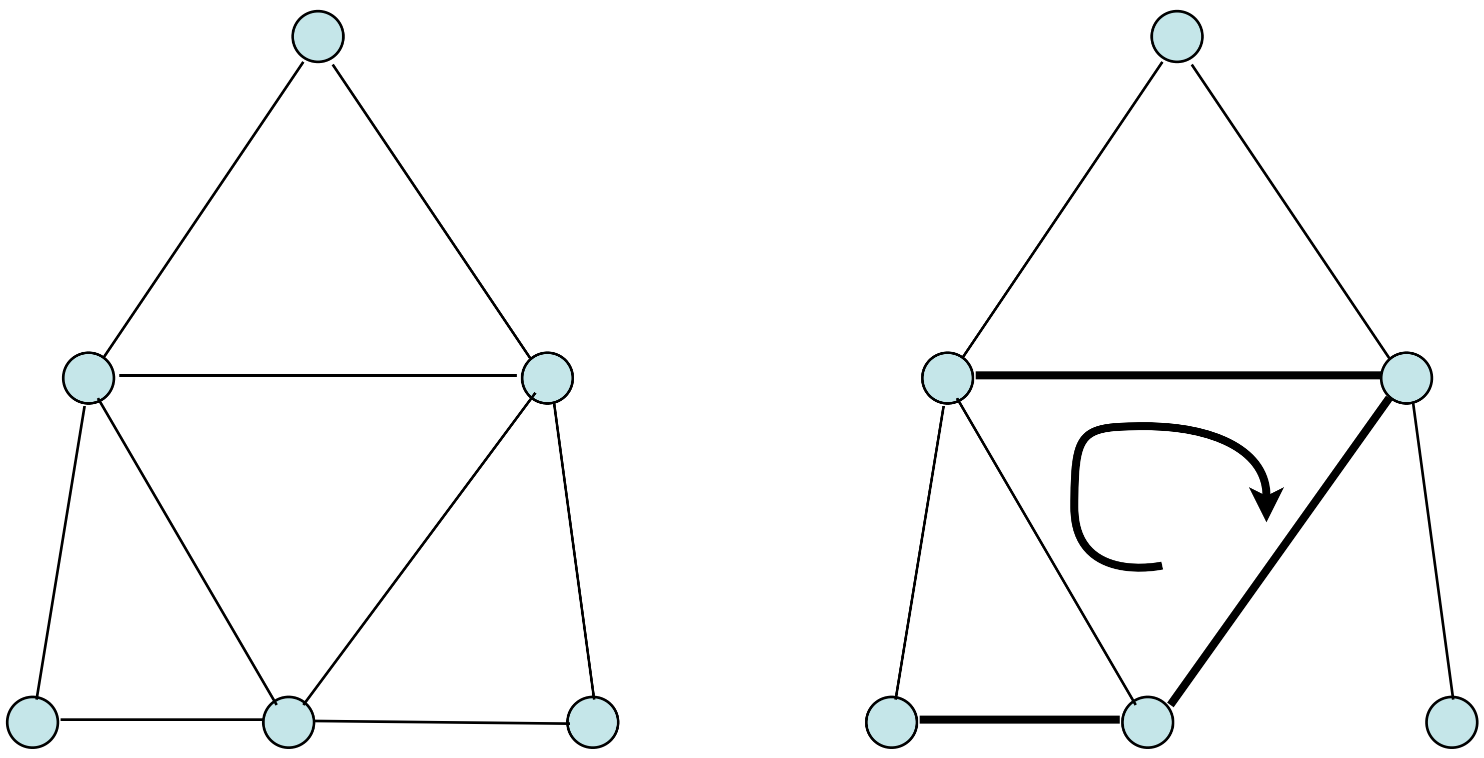
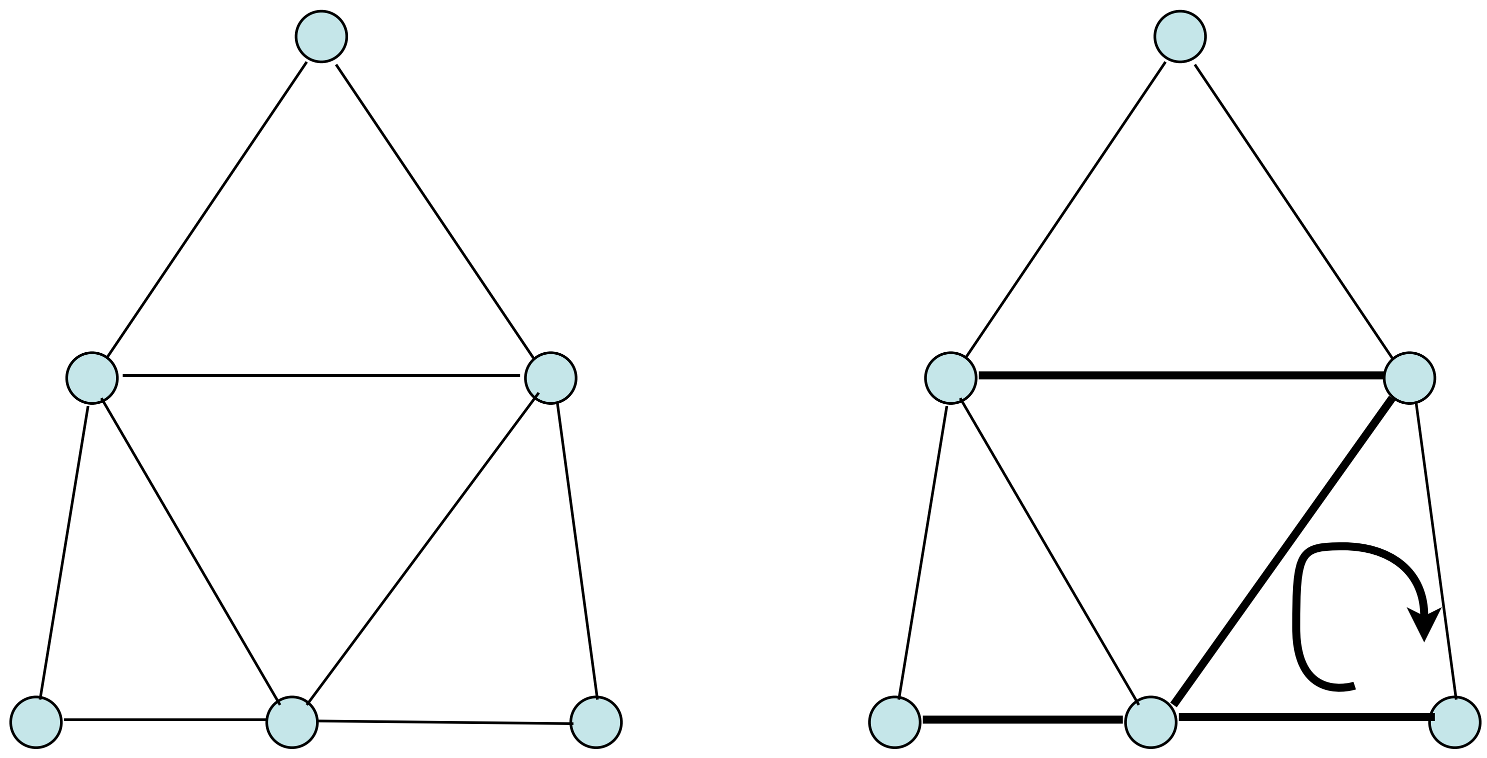
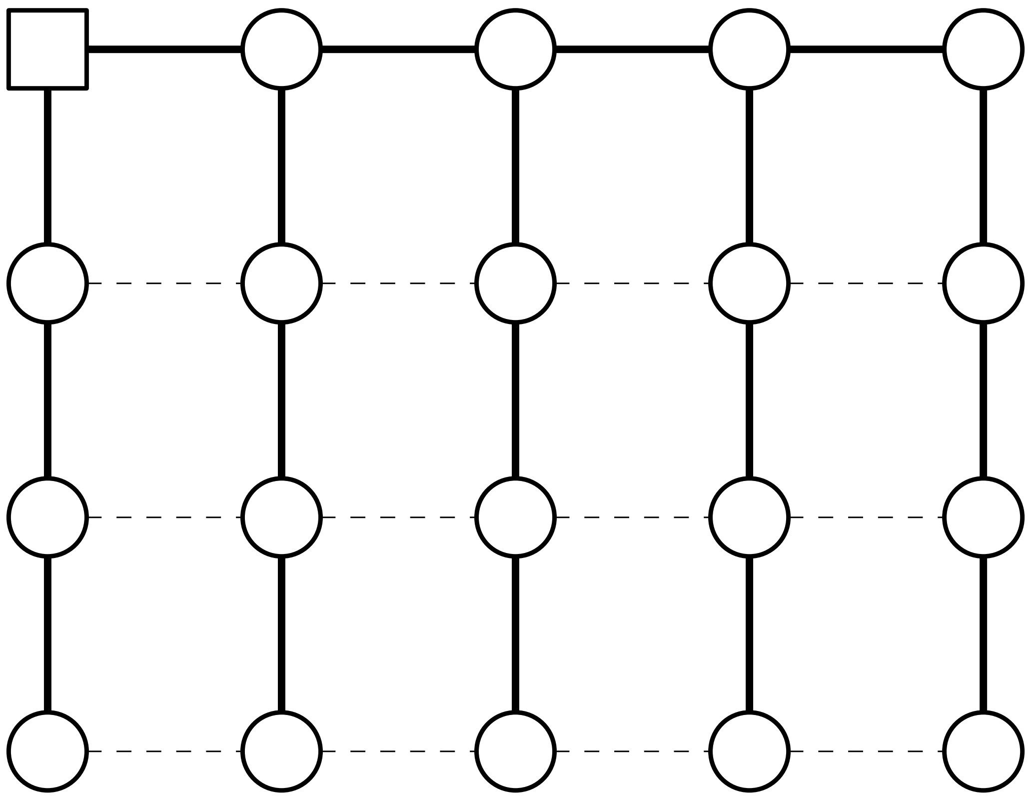


How to distribute the algorithm

Minimal cycles: harder to distribute but with smaller reconstruction error.



Simulation results on the 2D grid

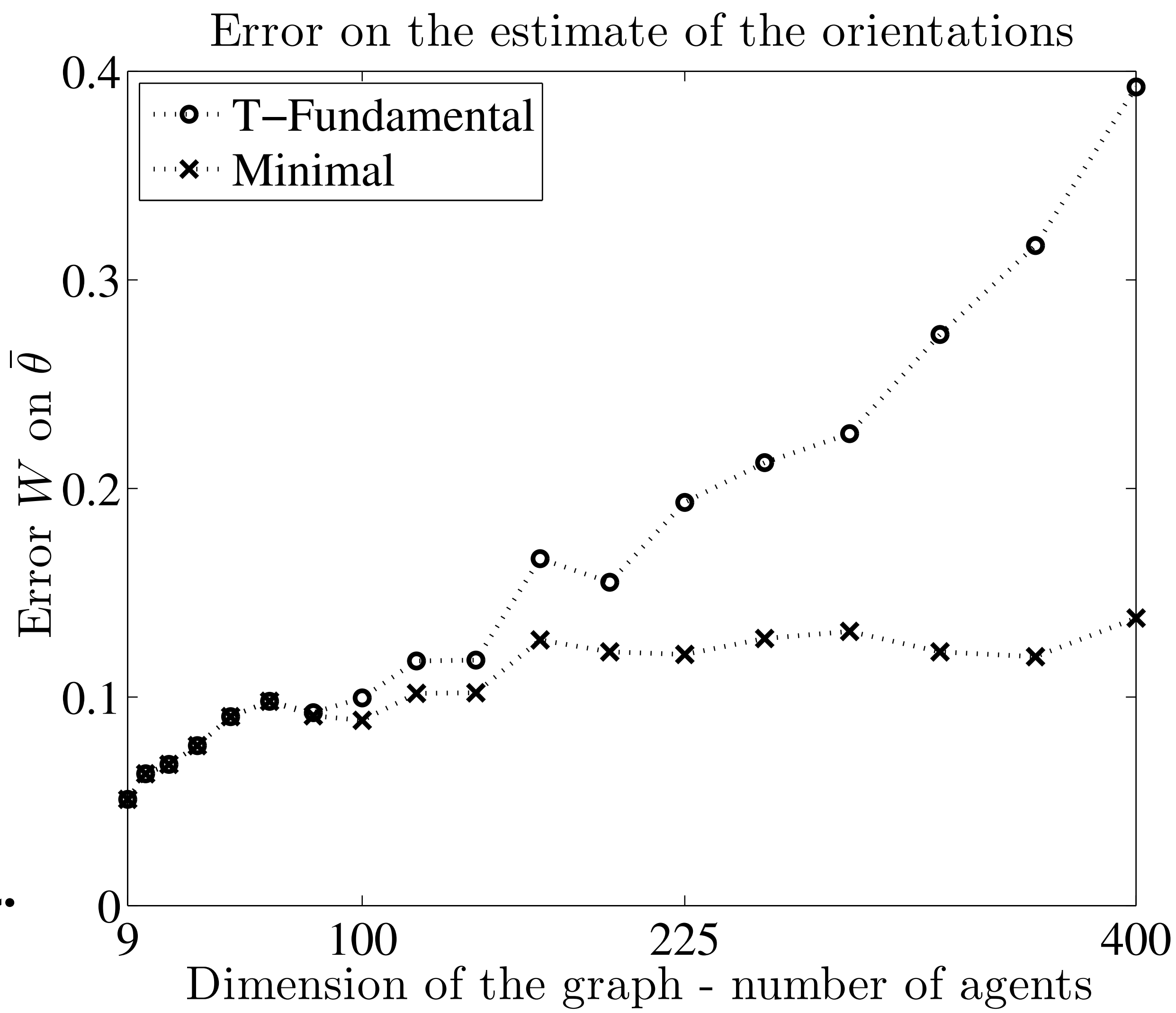


For this graph minimal cycles are shorter

the algorithm has stronger resilience w.r.t. the measurement noise

Estimation error

$$W = \frac{1}{N} || (\bar{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}})_{2\pi} ||^2$$



Open issues

- From the algorithmic point of view:
  - A better estimation algorithm for the vector of integers **K**.
  - A more distributed algorithms (asynchronous gossip type).
- From the performance analysis point of view:
  - Obtain estimates of the probability of error in the estimation of **K**.
- The time varying case in which the orientations vary in time (mobile cameras).
- Bayesian apporach in which there is an apriori knowledge that can be used.

Questions?