State-Aware Multiple Access for Networked Control Systems

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Outline

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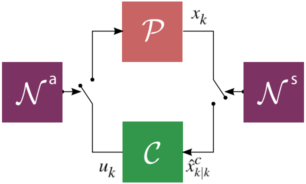
Networked Control Systems (NCSs)

- Networked Control Systems use a network to communicate between the sensors, controllers and actuators.
- Wireless is a broadcast medium:



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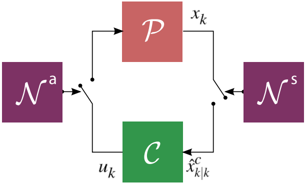

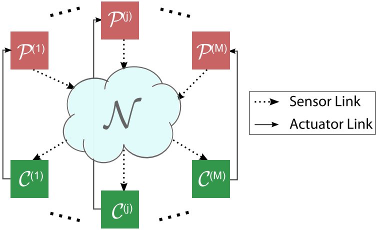

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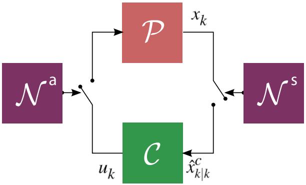

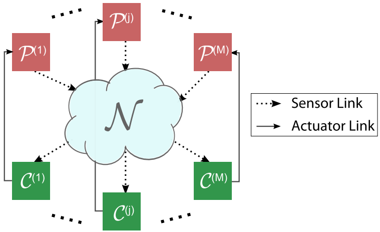

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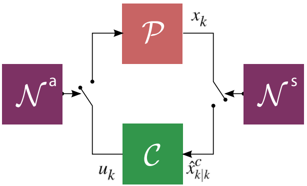

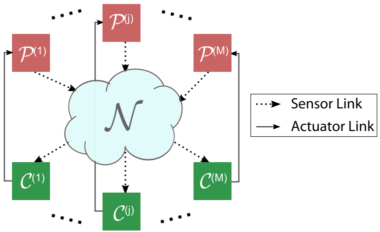

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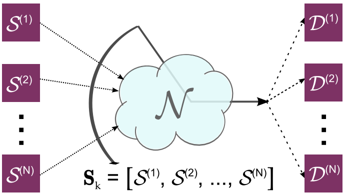
Multiple Access (MA) Protocols

Contention-free MA

- Transmissions guaranteed
- Requires scheduling
- Ill-suited to frequently changing networks



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- Collisions! Transmissions not guaranteed
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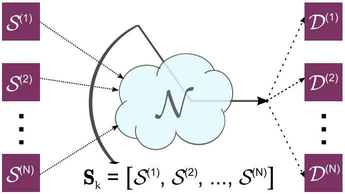

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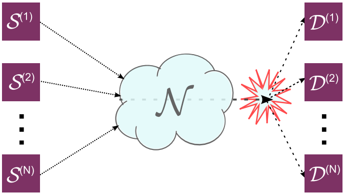

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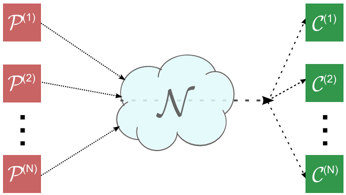
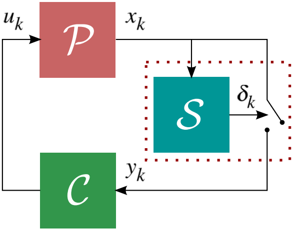


New MA for NCS

- Distributed mechanism
- Randomness in access is minimized
- Transmissions probabilistically guaranteed.

State-Aware MA

- Channel access depends on current state
- Realization of state-aware MA:
  - modifying existing protocols / introducing new protocols
  - regulating plant traffic



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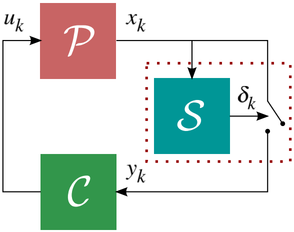

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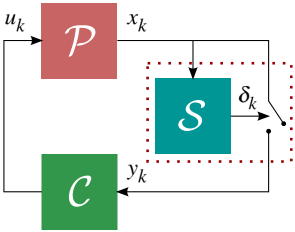

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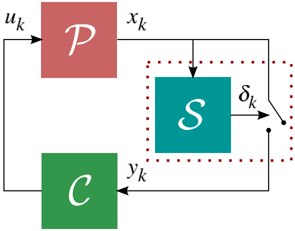

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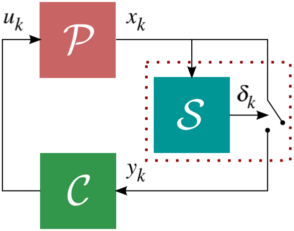

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- State-based Scheduler: local scheduler that selects packets to send to the MAC
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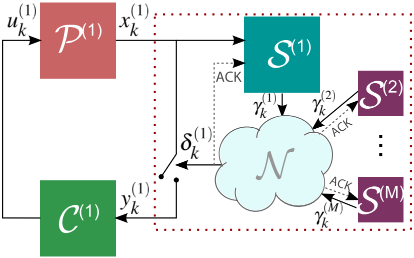
- RA determined by Contention Resolution Mechanism (CRM)



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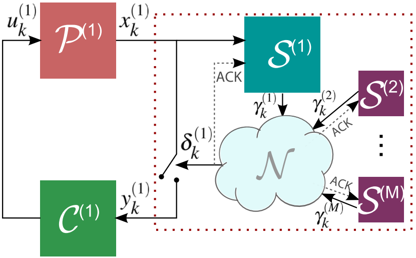

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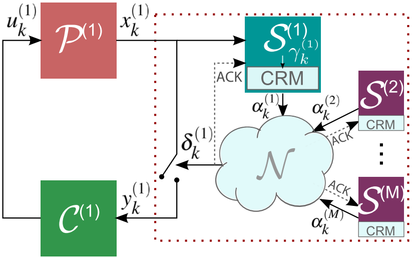

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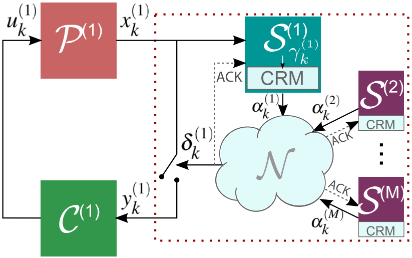

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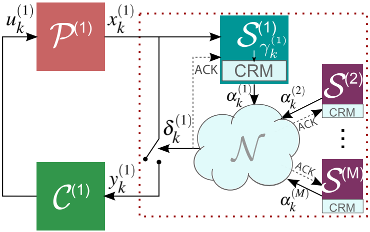
- Multiple Access on the Sensor Link



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- $\mathcal{C}$: Controller
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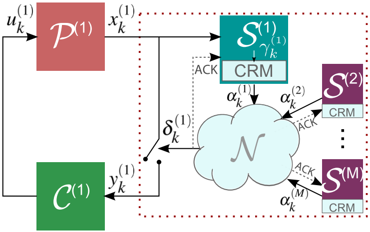

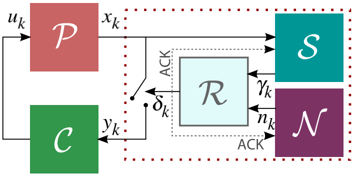

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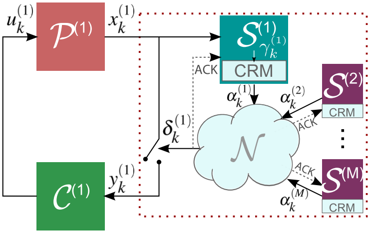

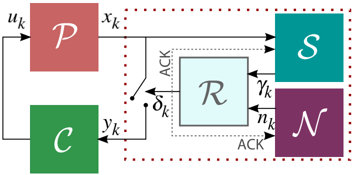

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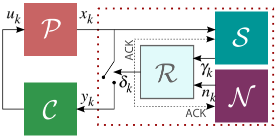
$$\gamma_k, n_k, \delta_k \in \{0, 1\}$$



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$$x_{k+1} = Ax_k + Bu_k + w_k \ ,$$
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- State-based Scheduler $\mathcal{S}$:

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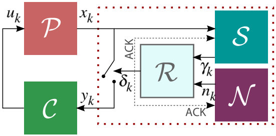

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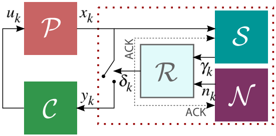

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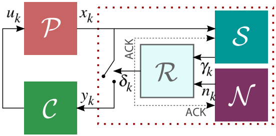

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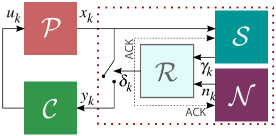

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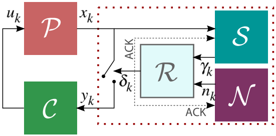

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- **Structural Analysis:**
  How does the architecture affect the closed-loop system and network performance?
- **Performance Analysis:**
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- **Stability Analysis:**
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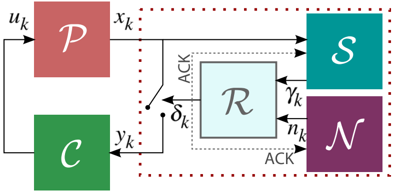

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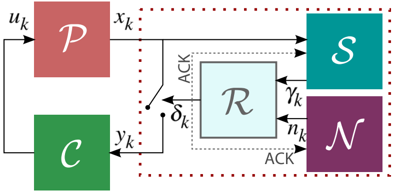

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Main Questions

How can we synthesize $\mathcal{S}$, $\mathcal{R}$ and $\mathcal{C}$ to achieve a certain performance?

- **Structural Analysis:**
  How does the architecture affect the closed-loop system and network performance?

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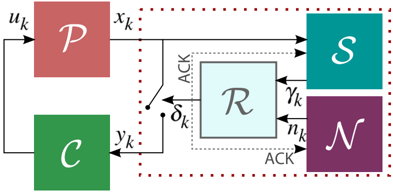
  How do the parameters affect stability of the closed-loop system?



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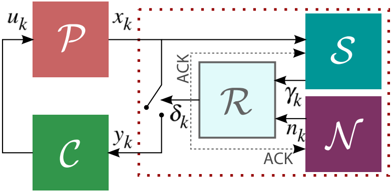

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Outline

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1. Introduction to NCS
2. Problem Formulation
3. Design of State-based Schedulers
   - Structural Analysis
   - Steady State Performance Analysis
   - Stability Analysis
4. Conclusions

Dual Effect Property

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Theorem

For the closed-loop system given by $\{\mathcal{P}, \mathcal{S}(f), \mathcal{C}(g)\}$, the control signal has a dual effect of order $r = 2$.

- Proof:
  - State Estimate:

$$\hat{x}_{k|k} = \begin{cases} x_k, & \delta_k = 1 \\ \mathbb{E}[x_k | \mathbb{I}_k^{\mathcal{C}}, \delta_k = 0], & \delta_k = 0 \end{cases}$$

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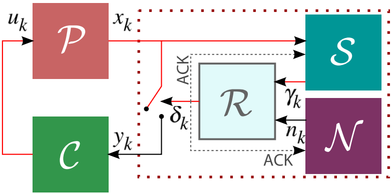

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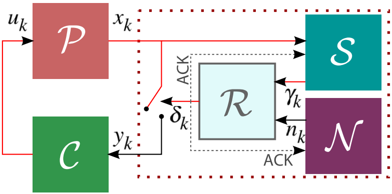

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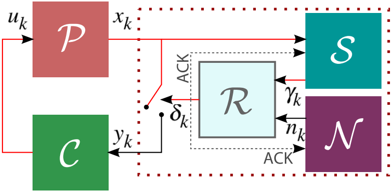

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Conditions for Certainty Equivalence

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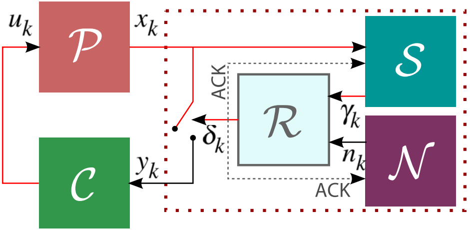
Corollary

The optimal controller for the system $\{\mathcal{P}, \mathcal{S}(f), \mathcal{C}(g)\}$, with respect to the cost J, is certainty equivalent if and only if the scheduling decisions are not a function of the applied controls.



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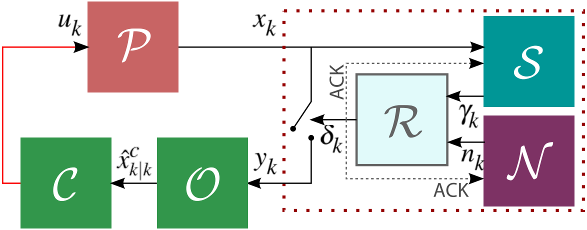

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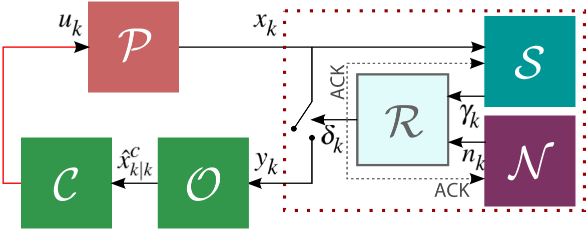

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Observer Design

- Estimate:

$$\hat{x}_{k|\tau_k} = A^{k-\tau_k} x_{\tau_k} + \sum_{s=1}^{k-\tau_k} A^{s-1} B u_{k-s} + \mathbb{E}\left[\sum_{s=1}^{k-\tau_k} A^{s-1} w_{k-s} \,\middle|\, \grave{f}_k, \ldots, \grave{f}_{\tau_k+1} = 0\right] \cdot \mathbb{P}\left(\gamma_k = 0 | \delta_k = 0\right)$$

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$$\gamma_k = f^{sym}\left(\sum_{s=1}^{k-\tau_{k-1}} A^{s-1} w_{k-s}\right);$$

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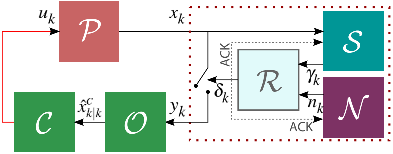

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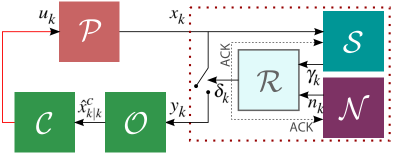

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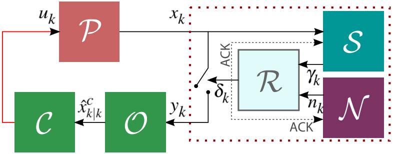

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Dual Predictor Architecture

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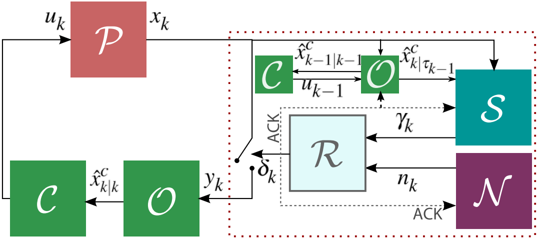
For the system $\{\mathcal{P}, \mathcal{S}, \mathcal{O}, \mathcal{C}\}$, using the dual predictor architecture results in a MMSE estimate and certainty equivalence.



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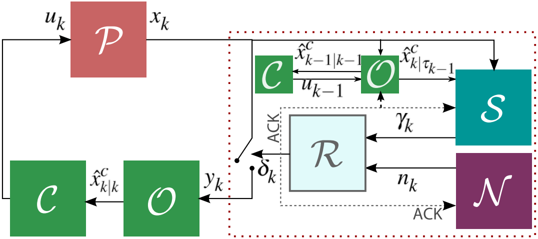

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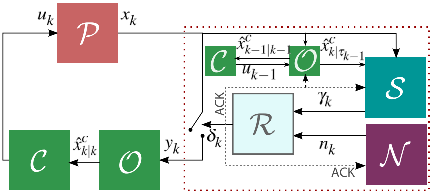
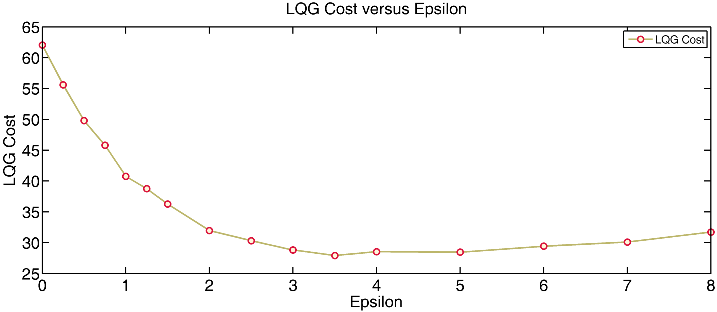
Simulation Results

- 20 scalar plants
- $A = 1$, $R_w = 1$ and $T = 10$
- $p_\alpha = \{1, 0.75, 0.5\}$
- $N = 10$, $Q_0 = Q_1 = Q_2 = 1$
- Best $\epsilon = 3.5$





Outline

Event-triggered and CRM Abstraction

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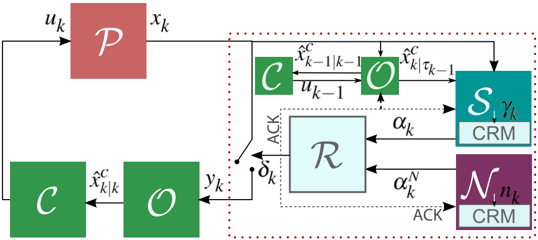

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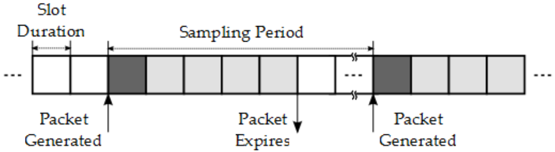

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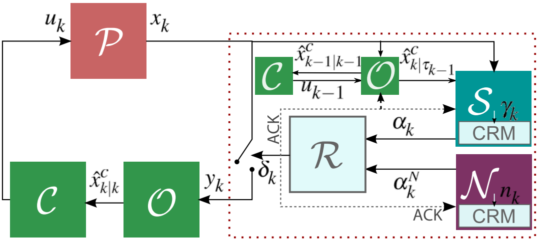

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- **Network-induced Correlation:**
  The scheduler output γ_k is correlated to the traffic n_k.
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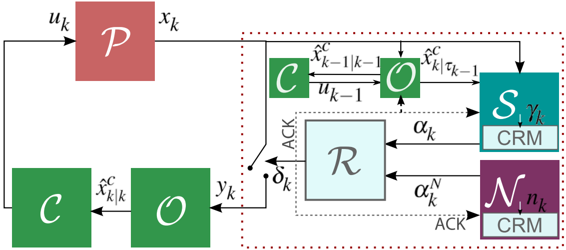

[Cervin and Henningsson, CDC 2008, Rabi and Johansson, ECC 2009]

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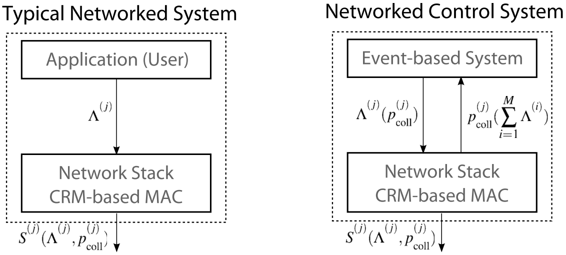

Joint CRM & Event-based Markov Model

- Innovations-based transitions to states (s, d)
- Index $d = min(d_{k-1}, F)$
- Index s:
  - $s = -1$: Idle
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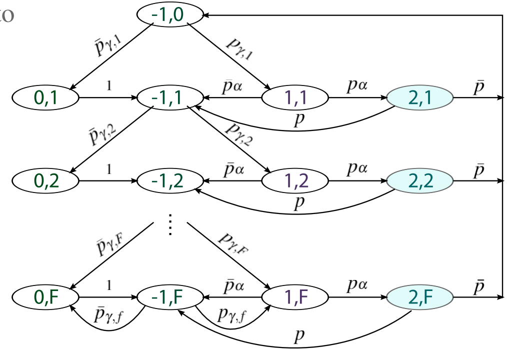
  - $s = 2$: Transmission
- $p_{\gamma,d}$: scheduler probability, p_α: p-persistence probability



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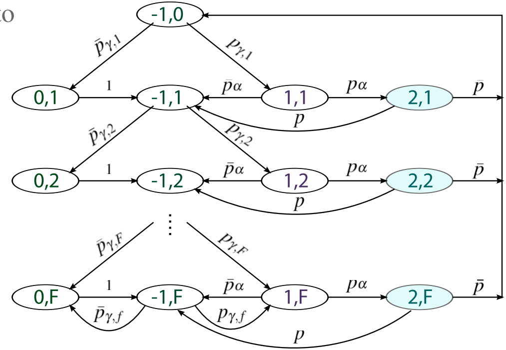

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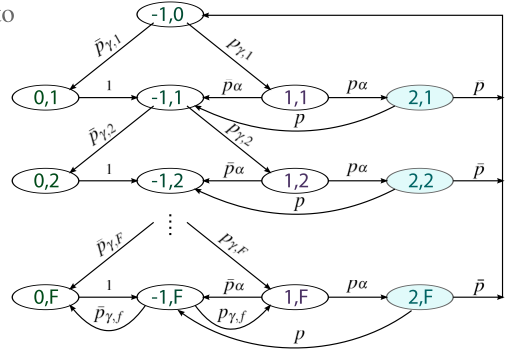

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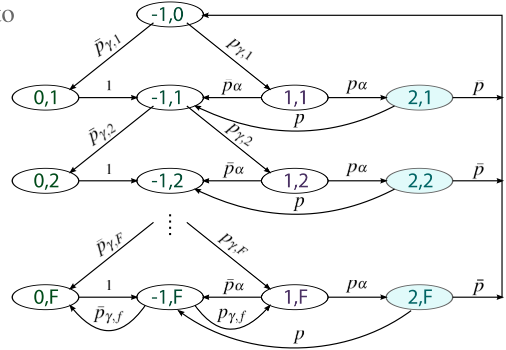

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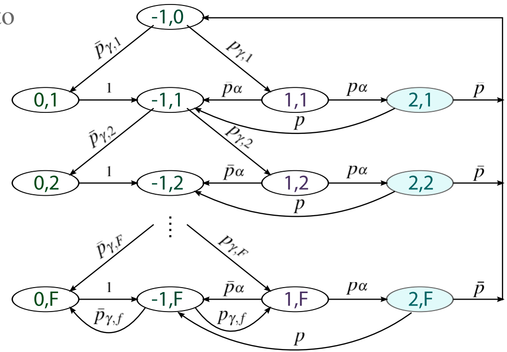

Key Assumption: Bianchi's conditional independence

The conditional probability of a busy channel for a node that attempts to transmit is given by an independent probability p for each node.

Steady State Performance Analysis

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Theorem

For the closed loop system given by $\{\mathcal{P}, \mathcal{S}(\tilde{f}), \mathcal{R}, \mathcal{C}\}$, the probability of a successful transmission in steady state is given by

$$\mathbb{P}(\delta_k^{(j)} = 1) = (1 - p^{(j)}) \cdot p_{TX}^{(j)} \,, \tag{1}$$

- Proof:
  - Sampling Instants: $\sum_{d=0}^{F} P^{(j)}_{(-1,d)} = 1$
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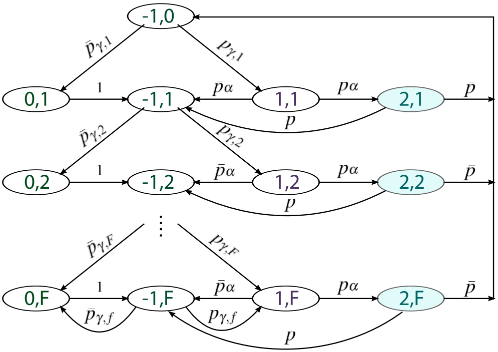

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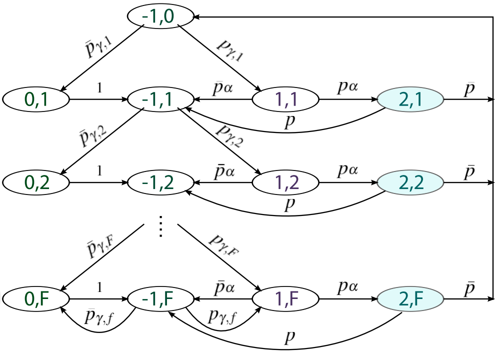

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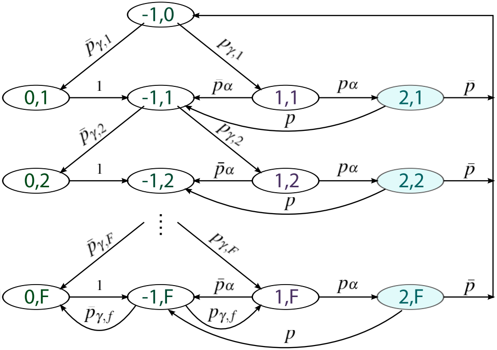

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Steady State Performance Analysis

Theorem

For the closed loop system given by $\{\mathcal{P}, \mathcal{S}(\tilde{f}), \mathcal{R}, \mathcal{C}\}$, the probability of a successful transmission in steady state is given by

$$\mathbb{P}(\delta_k^{(j)} = 1) = (1 - p^{(j)}) \cdot p_{TX}^{(j)} , \qquad (1)$$

- Proof:
  - Sampling Instants: $\sum_{d=0}^{F} p_{(-1,d)}^{(j)} = 1$
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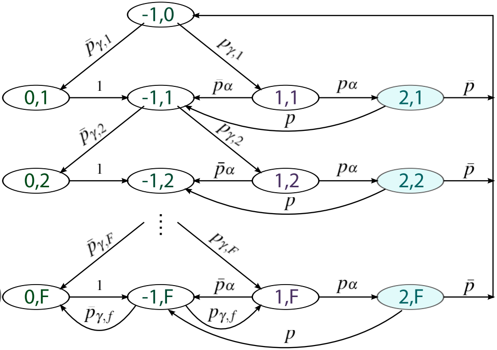

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Simulation Example

- $\mathcal{P}$: $x_{k+1} = x_k + u_k + w_k$, $w_k \sim \mathcal{N}(0, 1)$
- $\mathcal{S}$: $|x_k - x_{c,k}|^2 > \epsilon$

$$x_{c,k} = \begin{cases} \mathbb{E}[x_k | \mathbb{I}^C_{\tau_{k-1}}] & d_k < F \\ x_{k-F} & d_k \geq F \end{cases}$$

- Parameters: $M = 10$, $\epsilon = 1$, $p_\alpha = 0.2$, $R = 5$, $p_{\gamma,d} = \begin{bmatrix} 0.3171 & 0.5138 \end{bmatrix}$

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p_1	0.5937	0.5944
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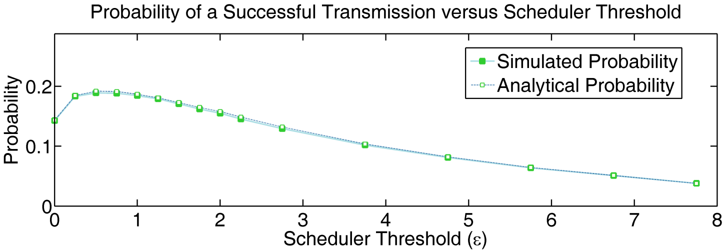
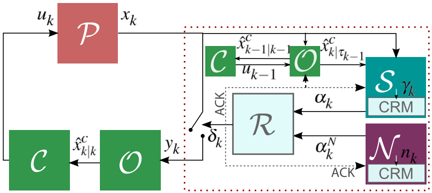


Outline

Lyapunov Mean Square Stability

- Let us consider infinite horizon LQG cost; we can now analyze stability of a closed-loop system in this network.



- Since Certainty Equivalence holds, we can translate the LMSS property from the state to the estimation error.
- There exists a constant ς, with $0 < \varsigma < \zeta$, such that the above condition is equivalent to $\limsup_{k\to\infty} \mathbb{E}[P_{k|k}] \leq \varsigma$.

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The state is said to possess mean square stability if given $\zeta > 0$, there exists $\xi(\zeta) > 0$ such that $|x_0| < \xi$ implies

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Markov Model

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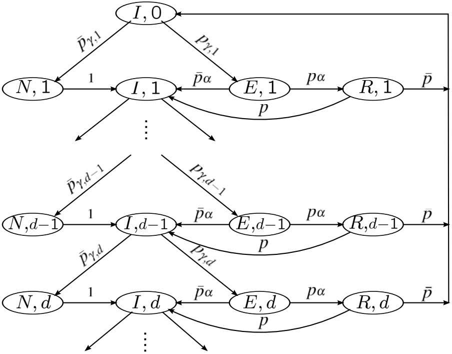
Assumptions:

- Bianchi's conditional probability holds
- Network is in steady state



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The network is said to be in steady state when the states (S, d), $\forall S \in \{I, N, E, R\}, d \geq 0$, are recurrent, or $p < 1$.

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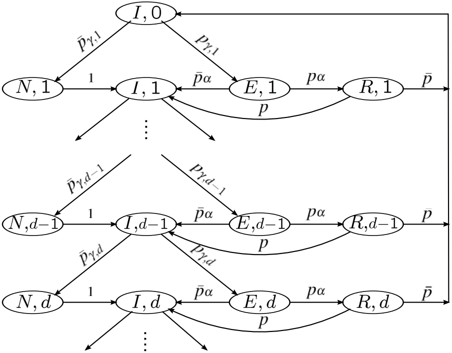

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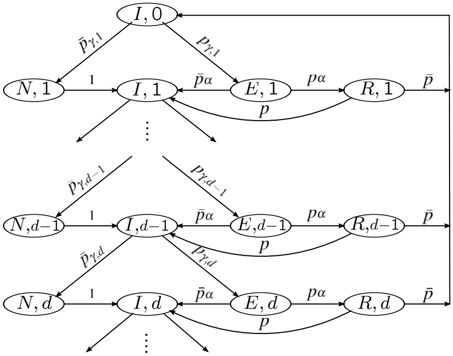
Upper Bound for Estimation Error

Estimation Error Covariance: $\mathbb{E}[P_{k|k}] = \sum_{d=0}^{\infty} P_d \, \mathbb{P}(d_k = d)$



Theorem: Upper Bound for Estimation Error in Idle State

$\hat{\phi}_{(I,d)} = \frac{1}{a}\hat{\phi}_{(I,d-1)} * \phi_N$, $\hat{\phi}_{(I,0)} = \phi_N$. Then, $\phi_{(I,d)} \succeq \hat{\phi}_{(I,d)} \; \forall d \geq 0$.

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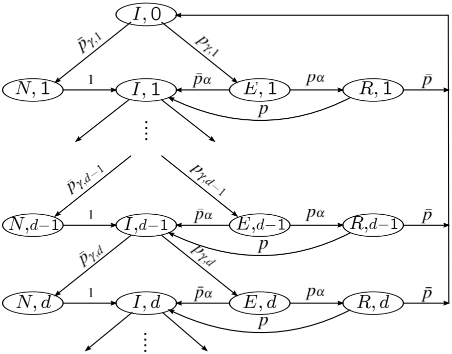

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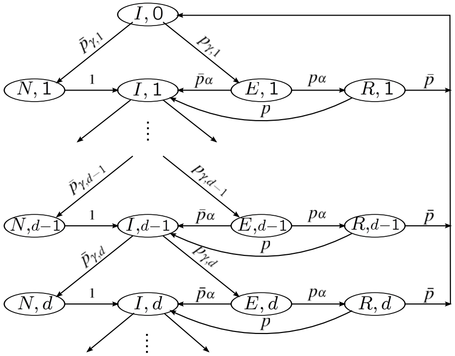

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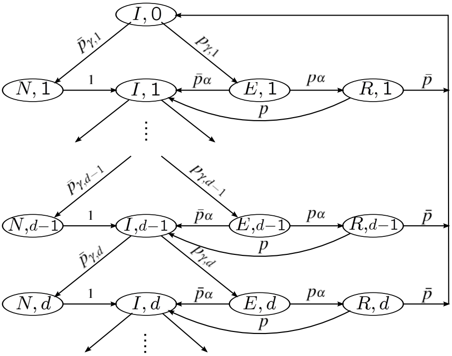

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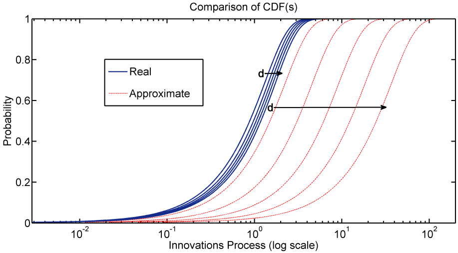

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Conditions for Stability

Theorem: Conditions for LMSS

Sufficient conditions for LMSS are given by

$$\limsup_{d\to\infty} \frac{p_{(I,d+1)}}{p_{(I,d)}} < \frac{1}{1+a^2}.$$

- **LMSS versus Steady State:**
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