Robust control of timed systems

Patricia Bouyer-Decitre

LSV, CNRS & ENS Cachan, France

Based on joint works with Nicolas Markey, Pierre-Alain Reynier and Ocan Sankur.
Acknowledgment to Nicolas and Ocan for slides.
Support from ERC project EQualIS.

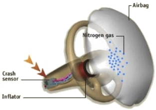
Outline

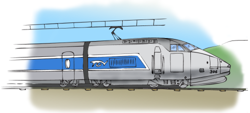
Time-dependent systems

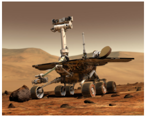
- We are interested in timed systems

Time-dependent systems

- We are interested in timed systems

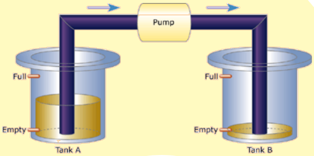






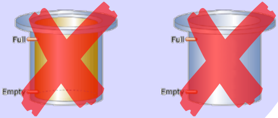
Model-checking and control

system:



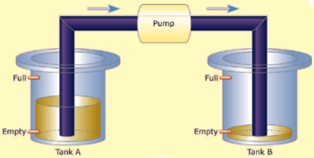
[http://www.embedded.com]

property:



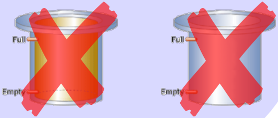
Model-checking and control

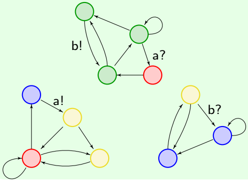
system:



[http://www.embedded.com]

property:

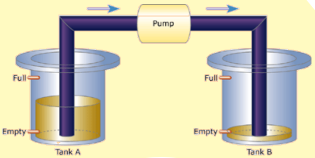




$$\mathbf{AG}(\neg \texttt{B.overfull} \wedge \neg \texttt{B.dried\_up})$$

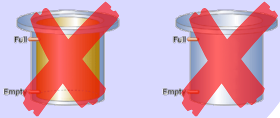
Model-checking and control

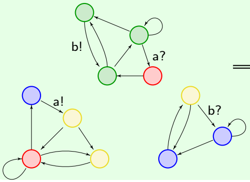
system:



[http://www.embedded.com]

property:



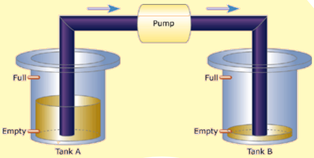


algorithm

$\mathbf{AG}(\neg \texttt{B.overfull} \wedge \neg \texttt{B.dried\_up})$

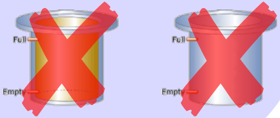
Model-checking and control

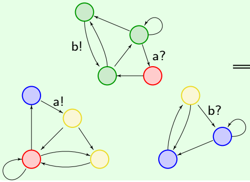
system:



[http://www.embedded.com]

property:





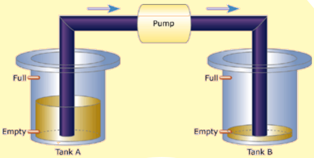
model-checking algorithm

AG(¬B.overfull ∧ ¬B.dried_up)

yes/no

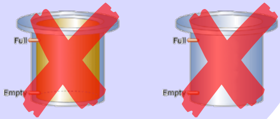
Model-checking and control

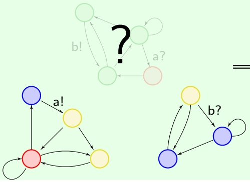
system:



[http://www.embedded.com]

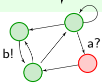
property:





control/synthesis algorithm

$\mathbf{AG}(\neg \texttt{B.overfull} \wedge \neg \texttt{B.dried\_up})$



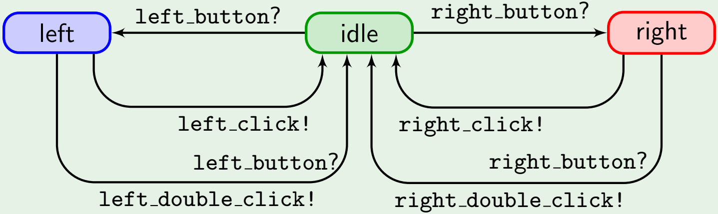
Reasoning about real-time systems

Timed automata [AD94]

A timed automaton is made of

- a finite automaton-based structure

Example (A computer mouse)



[AD94] Alur, Dill. A Theory of Timed Automata. *Theor. Comp. Science*, 1994.

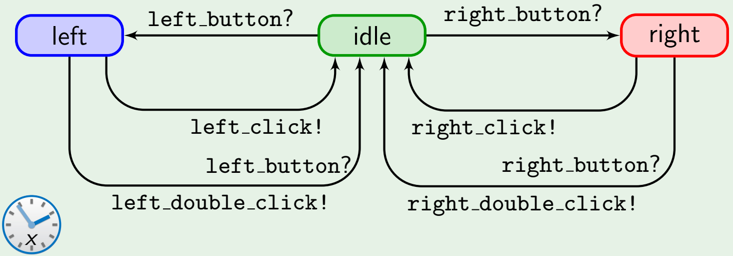
Reasoning about real-time systems

Timed automata [AD94]

A timed automaton is made of

- a finite automaton-based structure
- a set of clocks

Example (A computer mouse)



[AD94] Alur, Dill. A Theory of Timed Automata. *Theor. Comp. Science*, 1994.

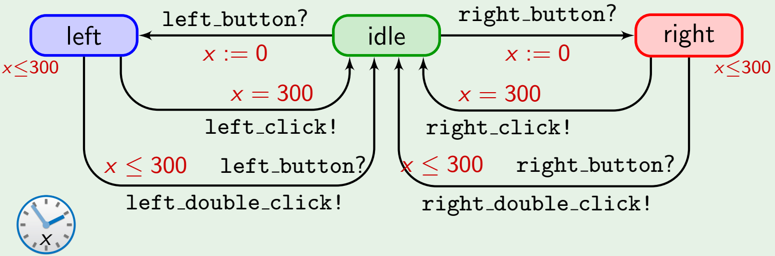
Reasoning about real-time systems

Timed automata [AD94]

A timed automaton is made of

- a finite automaton-based structure
- a set of clocks
- timing constraints on states and transitions

Example (A computer mouse)



[AD94] Alur, Dill. A Theory of Timed Automata. *Theor. Comp. Science*, 1994.

Discrete-time semantics

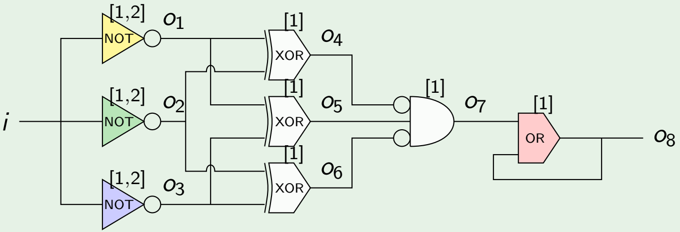
...because computers are digital!

[Alur91] Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

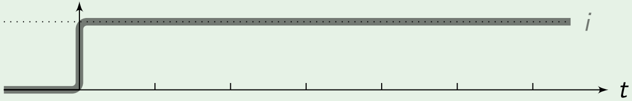
Discrete-time semantics

…because computers are digital!

Example [Alur91]



- under discrete-time, the output is always 0:

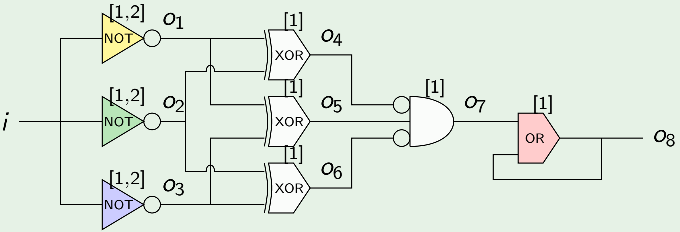


[Alur91] Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

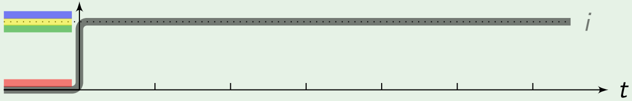
Discrete-time semantics

...because computers are digital!

Example [Alur91]



- under discrete-time, the output is always 0:

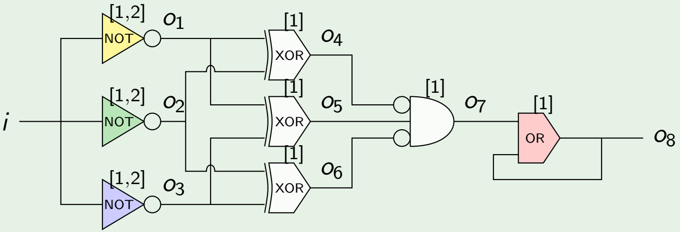


[Alur91] Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

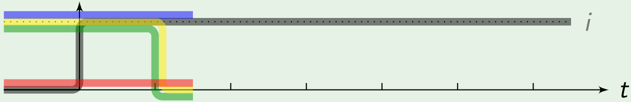
Discrete-time semantics

...because computers are digital!

Example [Alur91]



- under discrete-time, the output is always 0:

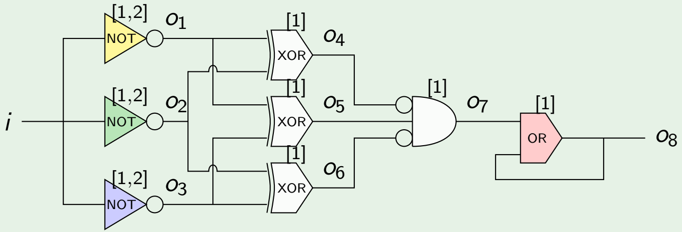


[Alur91] Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

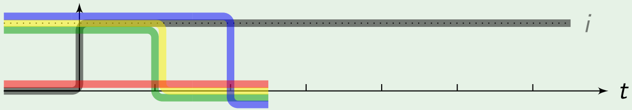
Discrete-time semantics

...because computers are digital!

Example [Alur91]



- under discrete-time, the output is always 0:

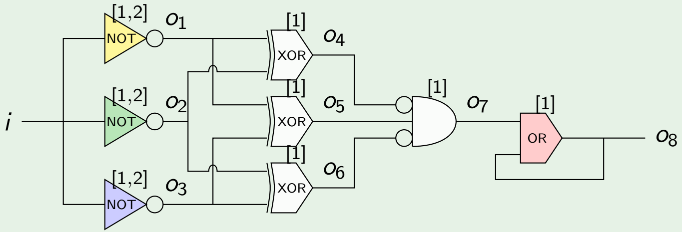


[Alur91] Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

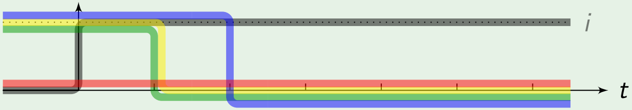
Discrete-time semantics

...because computers are digital!

Example [Alur91]



- under discrete-time, the output is always 0:

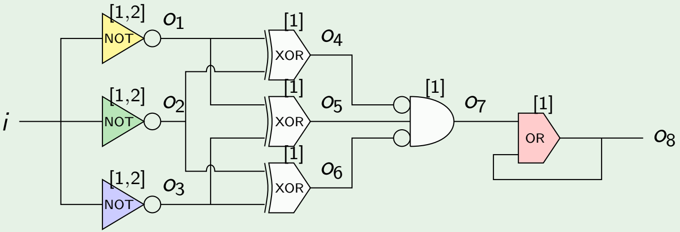


[Alur91] Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

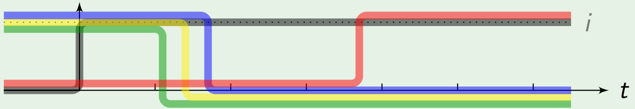
Discrete-time semantics

...because computers are digital!

Example [Alur91]



- under continuous-time, the output can be 1:



[Alur91] Techniques for automatic verification of real-time systems. *PhD thesis*, 1991.

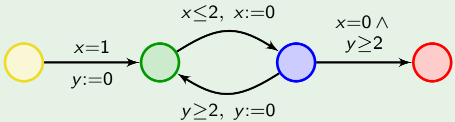
Continuous-time semantics

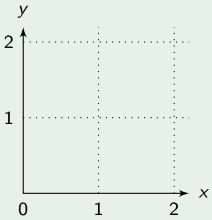
...real-time models for real-time systems!

Continuous-time semantics

...real-time models for real-time systems!

Example

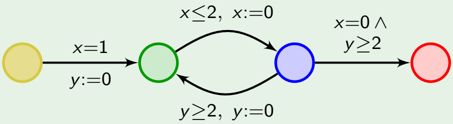


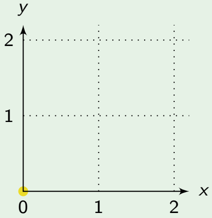


Continuous-time semantics

...real-time models for real-time systems!

Example

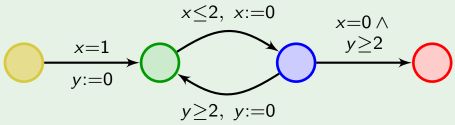


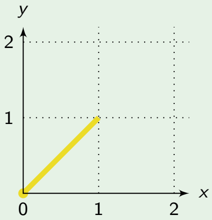


Continuous-time semantics

...real-time models for real-time systems!

Example

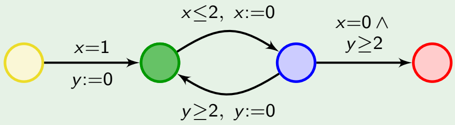


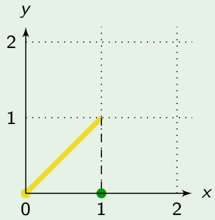


Continuous-time semantics

...real-time models for real-time systems!

Example

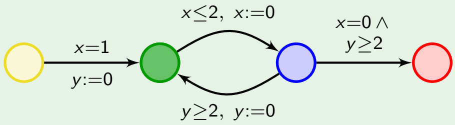


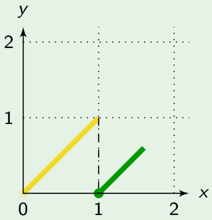


Continuous-time semantics

...real-time models for real-time systems!

Example

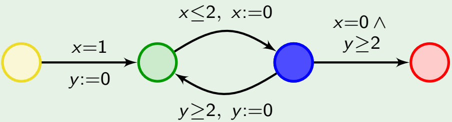


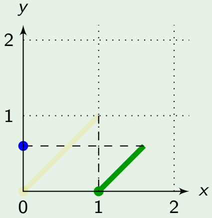


Continuous-time semantics

...real-time models for real-time systems!

Example

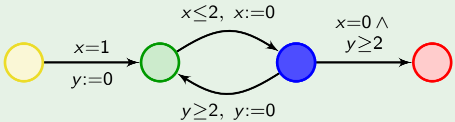


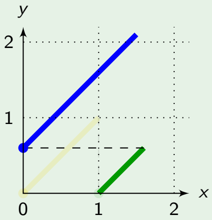


Continuous-time semantics

...real-time models for real-time systems!

Example

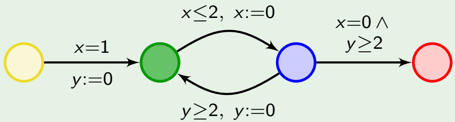


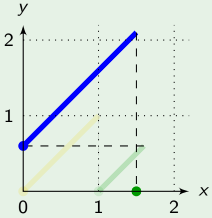


Continuous-time semantics

...real-time models for real-time systems!

Example

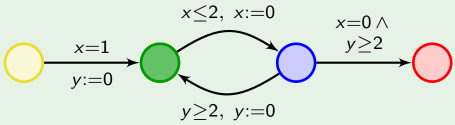


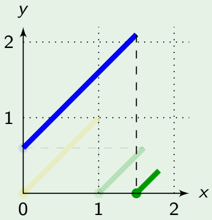


Continuous-time semantics

...real-time models for real-time systems!

Example

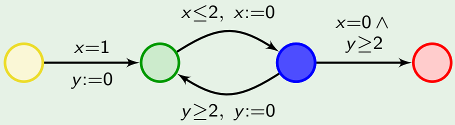


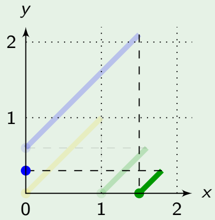


Continuous-time semantics

...real-time models for real-time systems!

Example

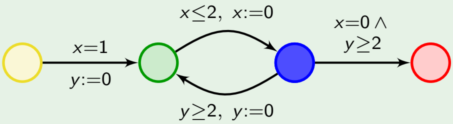


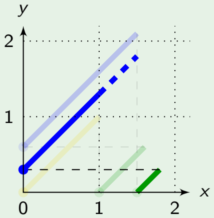


Continuous-time semantics

...real-time models for real-time systems!

Example

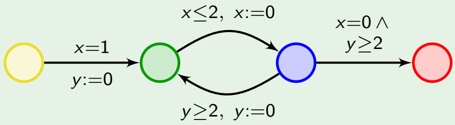


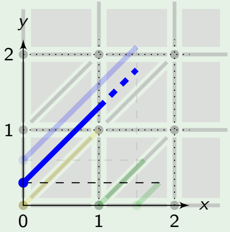


Continuous-time semantics

...real-time models for real-time systems!

Example

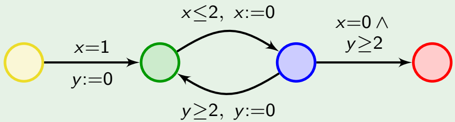


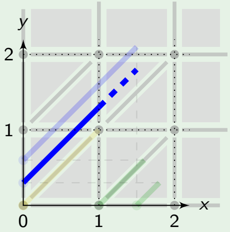


Continuous-time semantics

…real-time models for real-time systems!

Example





Theorem [AD94]

Reachability in timed automata is decidable (as well as many other important properties).

- Technical tool: region abstraction

Are we doing the right job?

The continuous-time semantics is
an idealization of a physical system.

Are we doing the right job?

The continuous-time semantics is
an idealization of a physical system.

- It might not be proper for implementation:
  - it assumes zero-delay transitions
  - it assumes infinite precision of the clocks
  - it assumes immediate communication between systems

Are we doing the right job?

The continuous-time semantics is
an idealization of a physical system.

- It might not be proper for implementation:
  - it assumes zero-delay transitions
  - it assumes infinite precision of the clocks
  - it assumes immediate communication between systems
- It may generate timing anomalies

Are we doing the right job?

The continuous-time semantics is an idealization of a physical system.

- It might not be proper for implementation:
  - it assumes zero-delay transitions
  - it assumes infinite precision of the clocks
  - it assumes immediate communication between systems
- It may generate timing anomalies
- It does not exclude non-realizable behaviours:
  - not only Zeno behaviours
  - many convergence phenomena are hidden

$\rightsquigarrow$ this requires infinite precision and might not be realizable

Are we doing the right job?

**The continuous-time semantics is
an idealization of a physical system.**

- It might not be proper for implementation:
  - it assumes zero-delay transitions
  - it assumes infinite precision of the clocks
  - it assumes immediate communication between systems
- It may generate timing anomalies
- It does not exclude non-realizable behaviours:
  - not only Zeno behaviours
  - many convergence phenomena are hidden

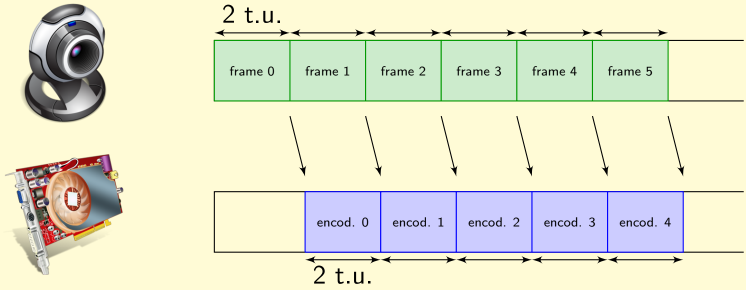
  $\rightsquigarrow$ this requires infinite precision and might not be realizable

Important questions

- Is the real system correct when it is proven correct on the model?
- Does actual work transfer to real-world systems? To what extent?

Example 1: Imprecision on clock values

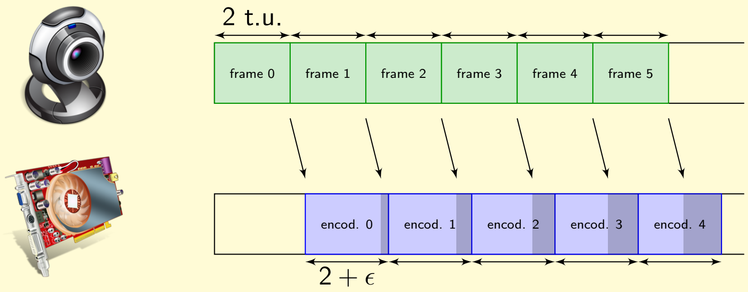
Frame capture [ACS10]



[ACS10] Abdellatif, Combaz, Sifakis. Model-based implementation of real-time applications. *Int. Conf. Embedded Software*, ACM 2010.

Example 1: Imprecision on clock values

Frame capture [ACS10]

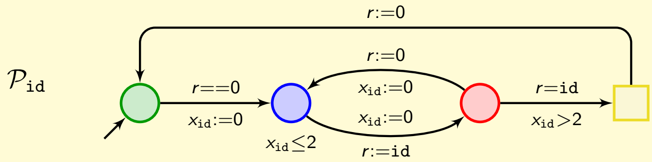


$\rightsquigarrow$ A frame will eventually be skipped

[ACS10] Abdellatif, Combaz, Sifakis. Model-based implementation of real-time applications. *Int. Conf. Embedded Software*, ACM 2010.

Example 2: Strict timing constraints

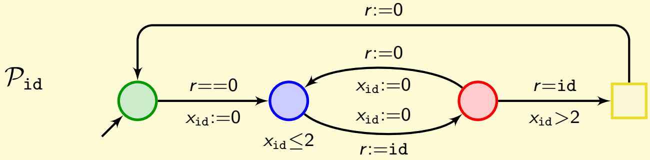
Mutual exclusion protocol [KLL+97]



[KLL+97] Kristoffersen, Laroussinie, Larsen, Pettersson, Yi. A compositional proof of a real-time mutual exclusion protocol. *TAPSOFT*, 1997.

Example 2: Strict timing constraints

Mutual exclusion protocol [KLL+97]

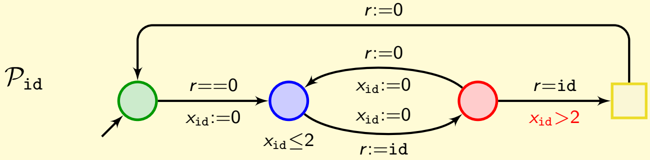


- When $\mathcal{P}_1$ and $\mathcal{P}_2$ run in parallel (sharing variable r), the state where both of them are in ☐ is not reachable.

[KLL+97] Kristoffersen, Laroussinie, Larsen, Pettersson, Yi. A compositional proof of a real-time mutual exclusion protocol. *TAPSOFT*, 1997.

Example 2: Strict timing constraints

Mutual exclusion protocol [KLL+97]



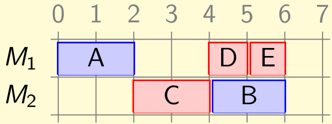
- When $\mathcal{P}_1$ and $\mathcal{P}_2$ run in parallel (sharing variable r), the state where both of them are in □ is not reachable.
- This property is lost when $x_{\mathtt{id}} > 2$ is replaced with $x_{\mathtt{id}} \geq 2$.

[KLL+97] Kristoffersen, Laroussinie, Larsen, Pettersson, Yi. A compositional proof of a real-time mutual exclusion protocol. *TAPSOFT*, 1997.

Example 3: Scheduling and timing anomaly

- Scheduling analysis with timed automata [AAM06]
- **Goal:** analyze a *work-conserving* scheduling policy on given scenarios (no machine is idle if a task is waiting for execution)

Example of a scenario



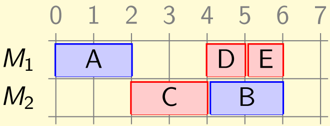
with the dependency constraints: $A \rightarrow B$ and $C \rightarrow D, E$.

1. A, D, E must be scheduled on machine M_1
2. B, C must be scheduled on machine M_2
3. C starts no sooner than 2 time units

[AAM06] Abdeddaim, Asarin, Maler. Scheduling with timed automata. *Theor. Comp. Science*, 2006.

Example 3: Scheduling and timing anomaly

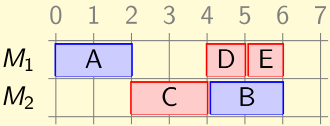
Example of a scenario



$\rightsquigarrow$ Schedulable in 6 time units

Example 3: Scheduling and timing anomaly

Example of a scenario

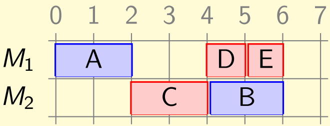


⇝ Schedulable in 6 time units

- Unexpectedly, the duration of A drops to 1.999

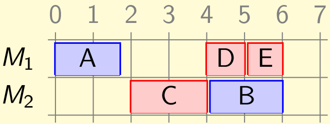
Example 3: Scheduling and timing anomaly

Example of a scenario



⇝ Schedulable in 6 time units

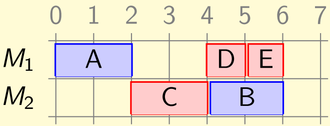
- Unexpectedly, the duration of A drops to 1.999



is not work-conserving

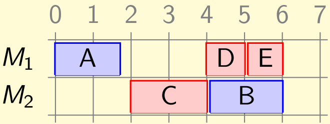
Example 3: Scheduling and timing anomaly

Example of a scenario

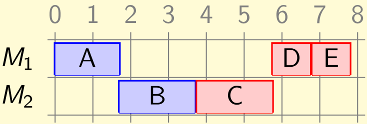


⤳ Schedulable in 6 time units

- Unexpectedly, the duration of A drops to 1.999



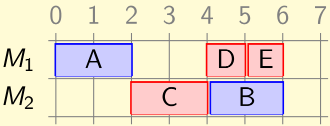
is not work-conserving



is work-conserving
and completes in 7.999 t.u.

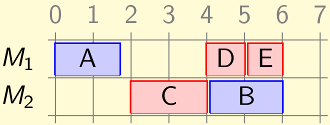
Example 3: Scheduling and timing anomaly

Example of a scenario

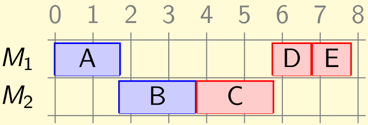


⇝ Schedulable in 6 time units

- Unexpectedly, the duration of A drops to 1.999



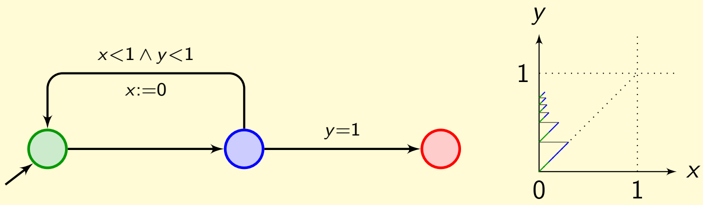
is not work-conserving



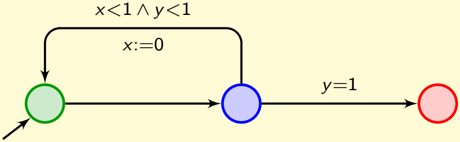
is work-conserving
and completes in 7.999 t.u.

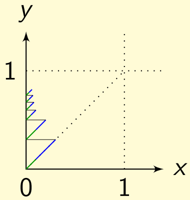
⇝ Standard analysis does not capture this **timing anomaly**

Example 4: Zeno behaviours



Example 4: Zeno behaviours

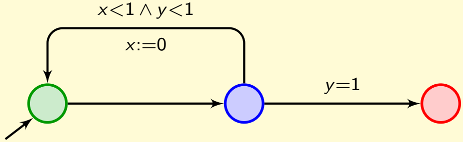


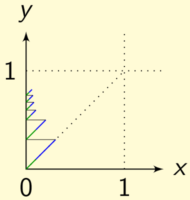


- Those are easy to detect and can be handled; [HS11]

[HS11] Herbreteau, Srivathsan. Coarse abstractions make Zeno behaviours difficult to detect, *Logic. Meth. Comp. Science*, 2011.

Example 4: Zeno behaviours

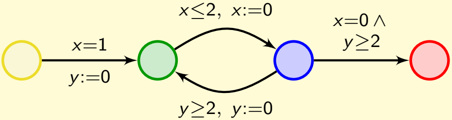




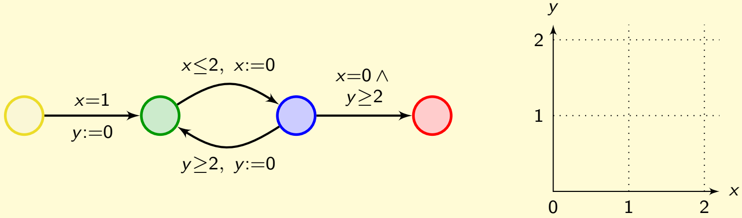
- Those are easy to detect and can be handled; [HS11]
- They are easy to remove by construction.

[HS11] Herbreteau, Srivathsan. Coarse abstractions make Zeno behaviours difficult to detect, *Logic. Meth. Comp. Science*, 2011.

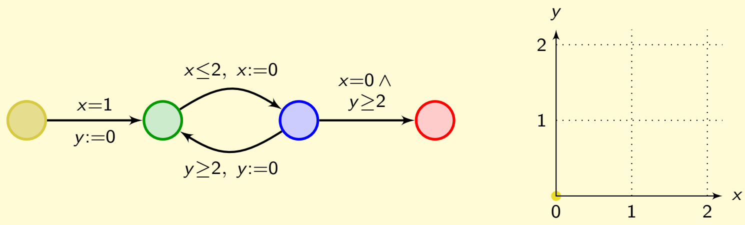
Example 5: More complex convergence phenomena



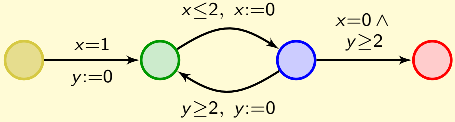
Example 5: More complex convergence phenomena

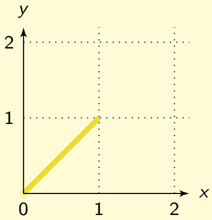


Example 5: More complex convergence phenomena

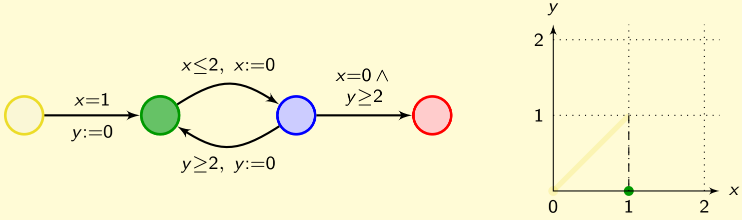


Example 5: More complex convergence phenomena

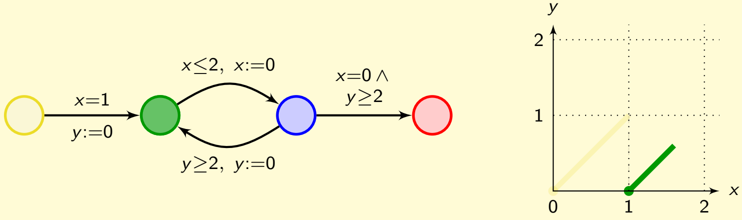




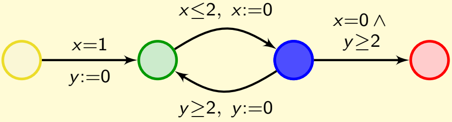
Example 5: More complex convergence phenomena

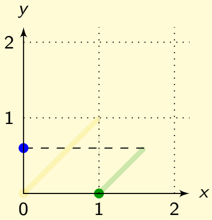


Example 5: More complex convergence phenomena

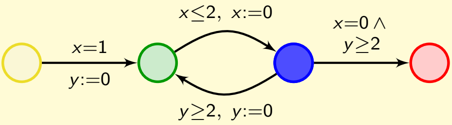


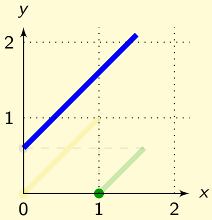
Example 5: More complex convergence phenomena



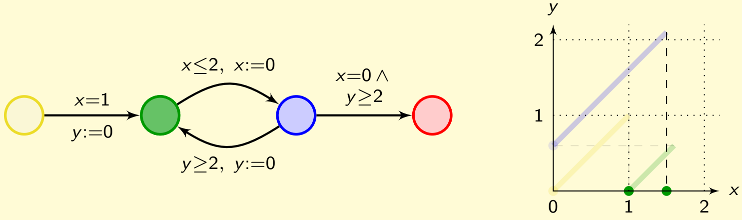


Example 5: More complex convergence phenomena

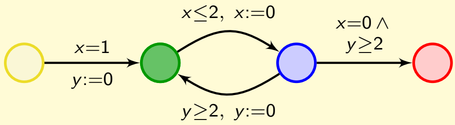


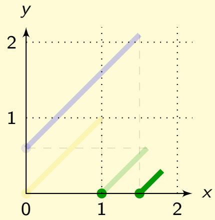


Example 5: More complex convergence phenomena

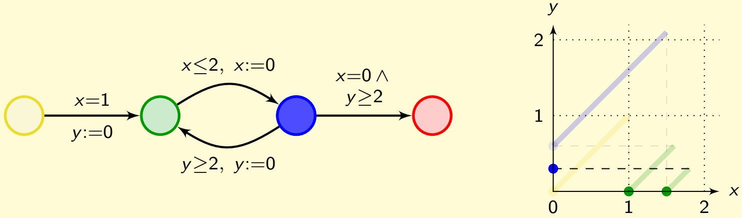


Example 5: More complex convergence phenomena

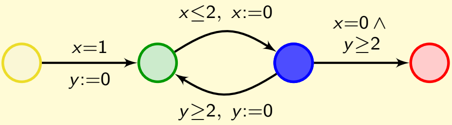


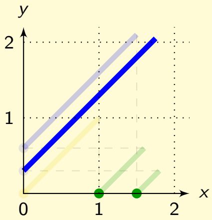


Example 5: More complex convergence phenomena

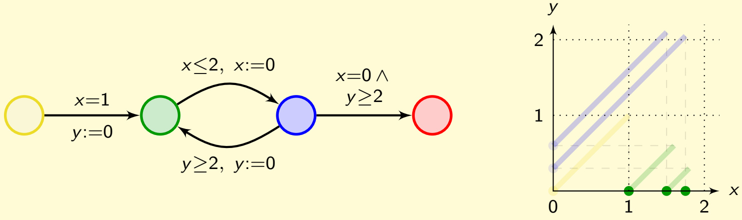


Example 5: More complex convergence phenomena

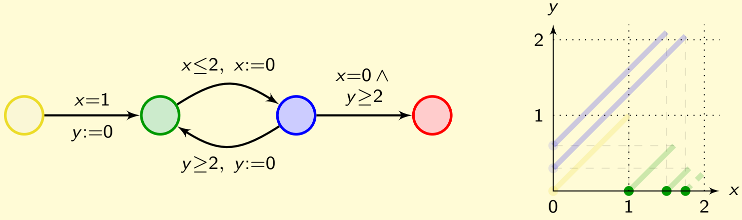




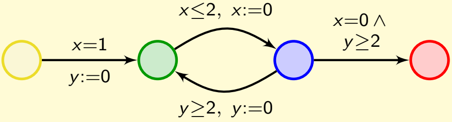
Example 5: More complex convergence phenomena

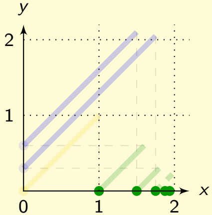


Example 5: More complex convergence phenomena



Example 5: More complex convergence phenomena





$\rightsquigarrow$ Value of clock x when hitting  is converging,
even though global time diverges

The goal

Add robustness to the theory of timed automata

The goal

Add robustness to the theory of timed automata

- We need to understand what is the real system behind the mathematical model, and also which implementation we have in mind, if any.

The goal

Add robustness to the theory of timed automata

- We need to understand what is the real system behind the mathematical model, and also which implementation we have in mind, if any.
- **Aim:** provide frameworks to build correct systems

The goal

Add robustness to the theory of timed automata

- We need to understand what is the real system behind the mathematical model, and also which implementation we have in mind, if any.
- **Aim:** provide frameworks to build robustly correct systems

The goal

Add robustness to the theory of timed automata

- We need to understand what is the real system behind the mathematical model, and also which implementation we have in mind, if any.
- **Aim:** provide frameworks to build robustly correct systems
  $\rightsquigarrow$ Robustness calls for specific theories for each application areas

The goal

Add robustness to the theory of timed automata

- We need to understand what is the real system behind the mathematical model, and also which implementation we have in mind, if any.
- **Aim:** provide frameworks to build robustly correct systems
  $\rightsquigarrow$ Robustness calls for specific theories for each application areas

Rest of the talk

We present a couple of frameworks that have been developed recently in this context

Outline

Robust “black-box” model-checking approach

Idea

Capture any real (or approximate) behaviours (e.g. the implementation) in the verification process

Robust "black-box" model-checking approach

Idea

Capture any real (or approximate) behaviours (e.g. the implementation) in the verification process

Due to imprecisions,

$$\text{"standard" correctness of } \mathcal{A} \quad \not\Rightarrow \quad \text{correctness of } \mathcal{A}_{\texttt{real}}$$

Robust "black-box" model-checking approach

Idea

Capture any real (or approximate) behaviours (e.g. the implementation) in the verification process

Due to imprecisions,

$$\text{“standard” correctness of } \mathcal{A} \quad \not\Rightarrow \quad \text{correctness of } \mathcal{A}_{\mathtt{real}}$$

$\rightsquigarrow$ We aim at proposing frameworks in which we will ensure the correctness of the real behaviour of the system

Robust "black-box" model-checking approach

Idea

Capture any real (or approximate) behaviours (e.g. the implementation) in the verification process

Due to imprecisions,

$$\text{"standard" correctness of } \mathcal{A} \quad \not\Rightarrow \quad \text{correctness of } \mathcal{A}_{\texttt{real}}$$

$\rightsquigarrow$ We aim at proposing frameworks in which we will ensure the correctness of the real behaviour of the system

We describe two such frameworks:

1. either we implement $\mathcal{A}$ and we prove:

$$\text{"robust" correctness of } \mathcal{A} \quad \Rightarrow \quad \text{correctness of } \mathcal{A}_{\texttt{real}}$$

Robust "black-box" model-checking approach

Idea

Capture any real (or approximate) behaviours (e.g. the implementation) in the verification process

Due to imprecisions,

$$\text{“standard” correctness of } \mathcal{A} \quad \not\Rightarrow \quad \text{correctness of } \mathcal{A}_{\texttt{real}}$$

$\rightsquigarrow$ We aim at proposing frameworks in which we will ensure the correctness of the real behaviour of the system

We describe two such frameworks:

1. either we implement $\mathcal{A}$ and we prove:

$$\text{“robust” correctness of } \mathcal{A} \quad \Rightarrow \quad \text{correctness of } \mathcal{A}_{\texttt{real}}$$

2. or we build and implement $\mathcal{B}$, and we prove:

$$\text{correctness of } \mathcal{A} \quad \Rightarrow \quad \text{“robust” correctness of } \mathcal{B} \quad \Rightarrow \quad \text{correctness of } \mathcal{B}_{\texttt{real}}$$

Outline

Parameterized enlarged semantics for timed automata

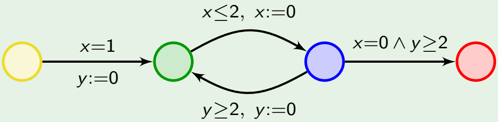
A transition can be taken at any time in $[t-\delta; t+\delta]$

Parameterized enlarged semantics for timed automata

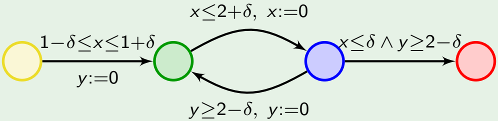
A transition can be taken at any time in $[t - \delta; t + \delta]$

Example

Given a parameter δ,



is transformed into



Parameterized enlarged semantics – Discussion

What is the relevance of this semantics?

- This is a worst-case approach
- This captures approximate behaviours of the system
- One can define program semantics such that for every $\epsilon > 0$:

$$\mathcal{A} \subseteq \mathtt{program}_{\epsilon}(\mathcal{A}) \subseteq \mathcal{A}_{f(\epsilon)}$$

ϵ: parameters of the semantics

[DDR04] De Wulf, Doyen, Raskin. Almost ASAP semantics: From timed models to timed implementations *HSCC*, 2004.
[SBM11] Sankur, Bouyer, Markey. Shrinking Timed Automata. *FSTTCS*, 2011.

Parameterized enlarged semantics – Discussion

What is the relevance of this semantics?

- This is a worst-case approach
- This captures approximate behaviours of the system
- One can define program semantics such that for every $\epsilon > 0$:

$$\mathcal{A} \subseteq \mathtt{program}_\epsilon(\mathcal{A}) \subseteq \mathcal{A}_{f(\epsilon)}$$

ϵ: parameters of the semantics

Methodology

- Design $\mathcal{A}$
- Verify $\mathcal{A}_\delta$ (better if δ is a parameter)
- Implement $\mathcal{A}$

Parameterized enlarged semantics – Discussion

What is the relevance of this semantics?

- This is a worst-case approach
- This captures approximate behaviours of the system
- One can define program semantics such that for every $\epsilon > 0$:

$$\mathcal{A} \subseteq \mathtt{program}_{\epsilon}(\mathcal{A}) \subseteq \mathcal{A}_{f(\epsilon)}$$

ϵ: parameters of the semantics

Methodology

- Design $\mathcal{A}$
- Verify $\mathcal{A}_\delta$ (better if δ is a parameter)
- Implement $\mathcal{A}$

$\rightsquigarrow$ This is good for designing systems with simple timing constraints (e.g. equalities).

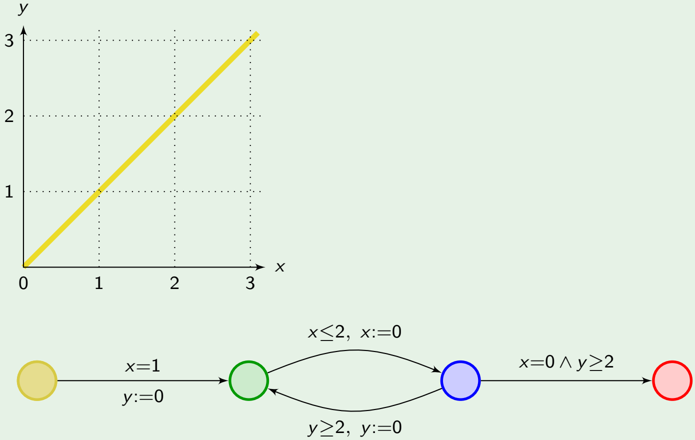
Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

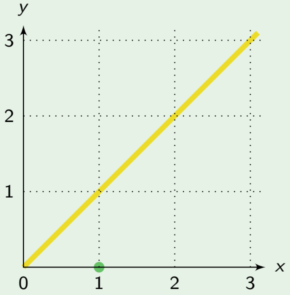
Example

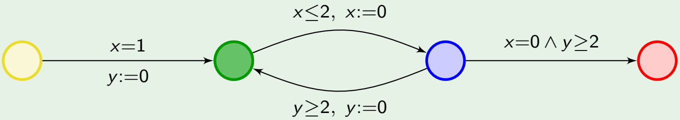


Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

Example

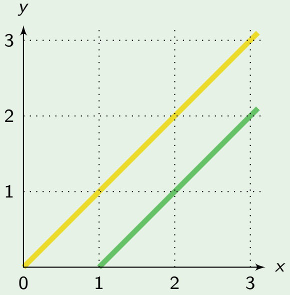


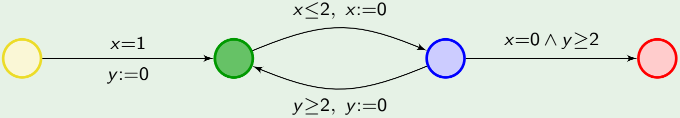


Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

Example

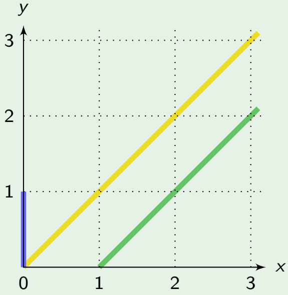


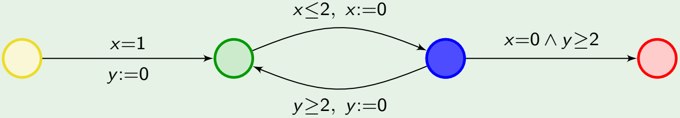


Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

Example

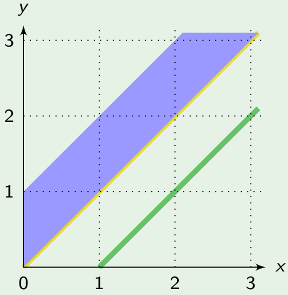


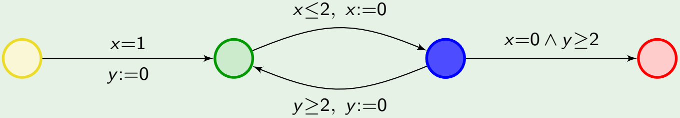


Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

Example

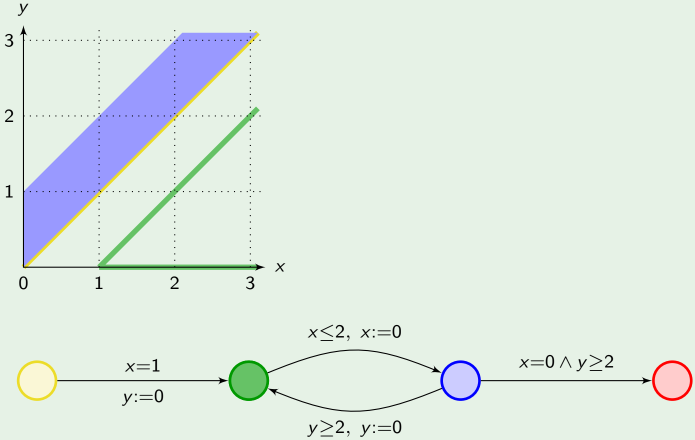




Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

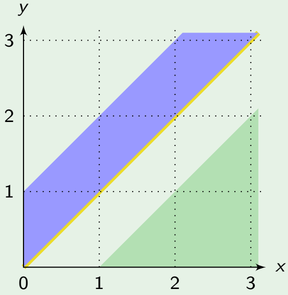
Example

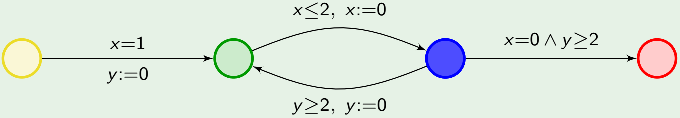


Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

Example

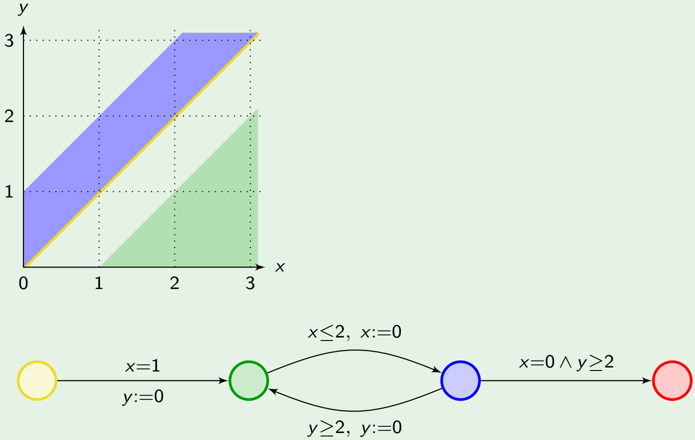




Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

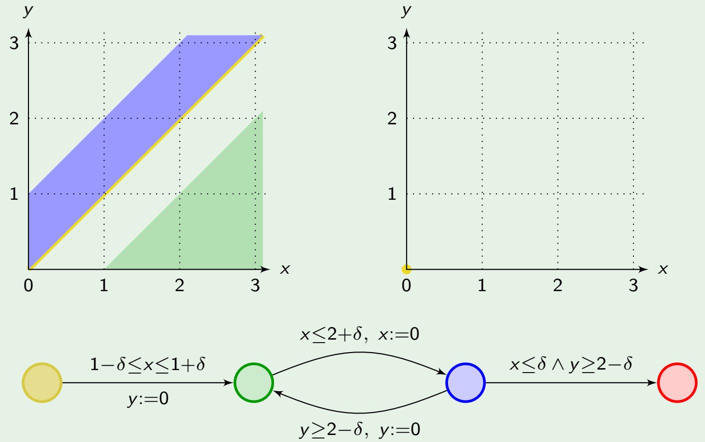
Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

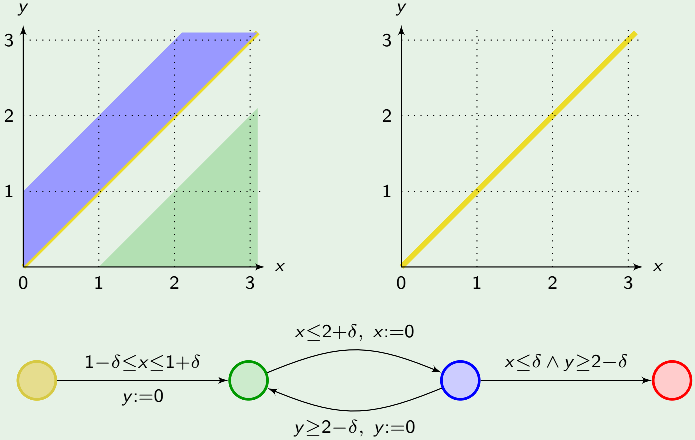
Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

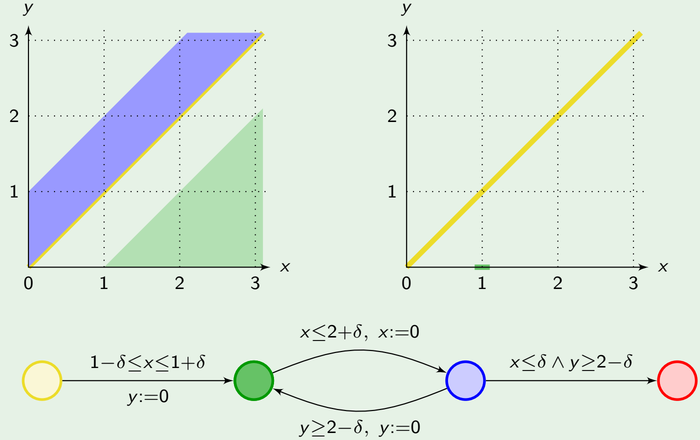
Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

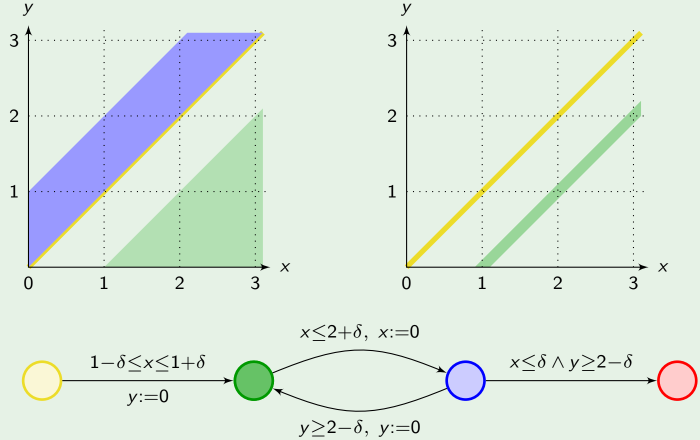
Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

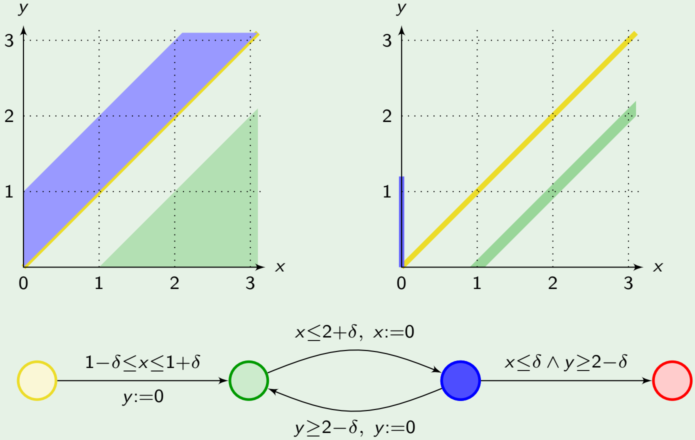
Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

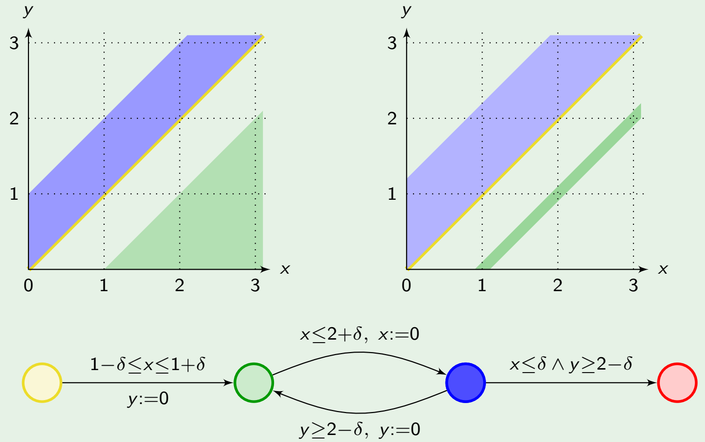
Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

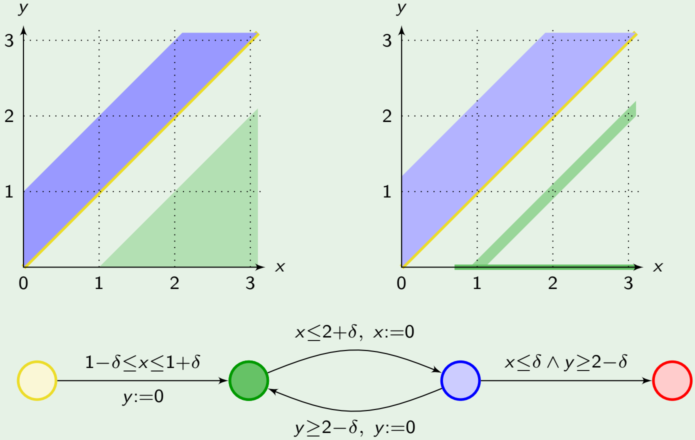
Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

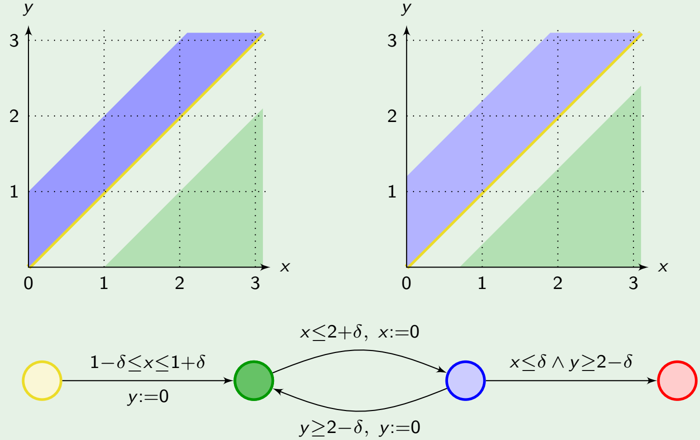
Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

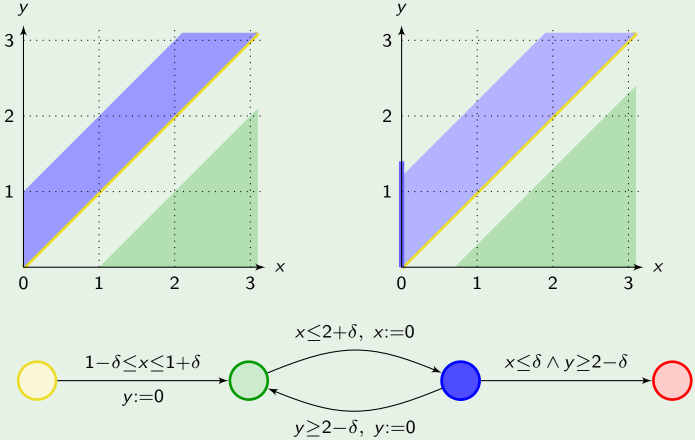
Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

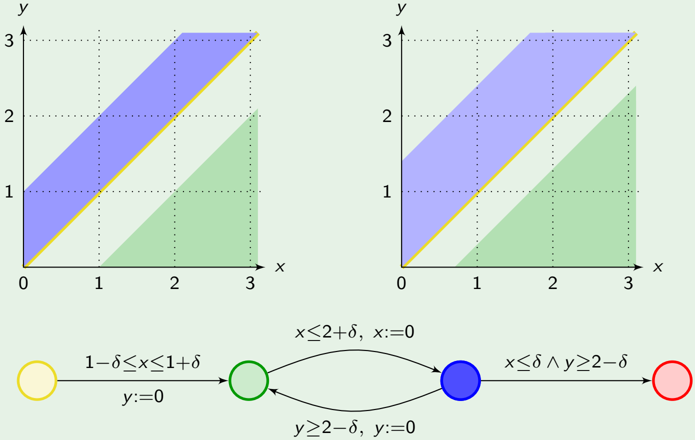
Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

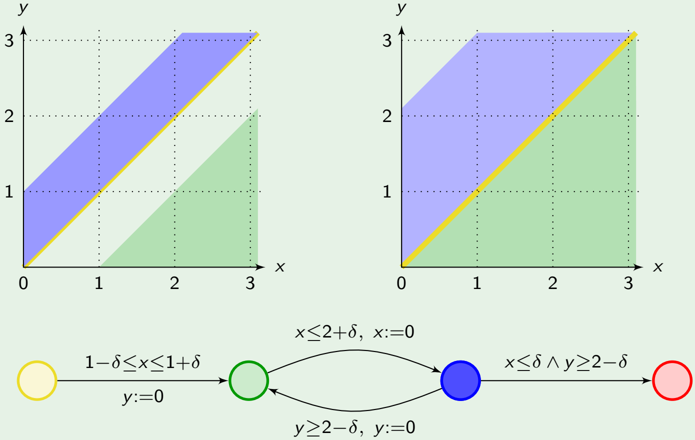
Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

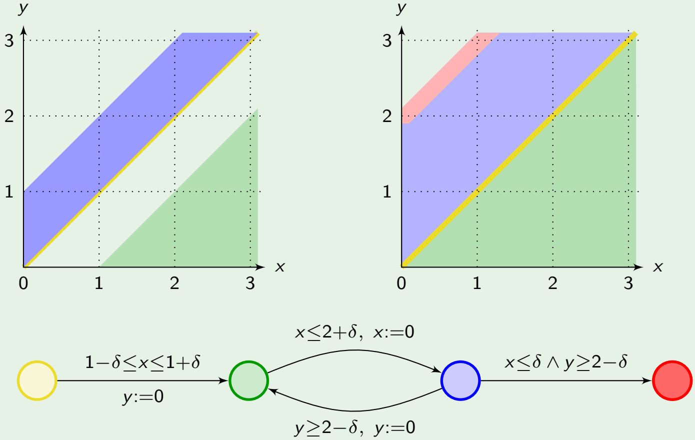
Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

Example



Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

The (parameterized) robust model-checking problem

It asks whether there is some $\delta_0 > 0$ such that for every $0 \leq \delta \leq \delta_0$, $\mathcal{A}_\delta \models \varphi$.

Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

The (parameterized) robust model-checking problem

It asks whether there is some $\delta_0 > 0$ such that for every $0 \leq \delta \leq \delta_0$, $\mathcal{A}_\delta \models \varphi$.

- When δ is small, truth of φ is independent of δ

Parameterized enlarged semantics – Algorithmics

$\leadsto$ It adds extra behaviours, however small may be parameter δ

The (parameterized) robust model-checking problem

It asks whether there is some $\delta_0 > 0$ such that for every $0 \leq \delta \leq \delta_0$, $\mathcal{A}_\delta \models \varphi$.

- When δ is small, truth of φ is independent of δ
- It can be computed using a simple extension of the region automaton

Parameterized enlarged semantics – Algorithmics

$\rightsquigarrow$ It adds extra behaviours, however small may be parameter δ

The (parameterized) robust model-checking problem

It asks whether there is some $\delta_0 > 0$ such that for every $0 \leq \delta \leq \delta_0$, $\mathcal{A}_\delta \models \varphi$.

- When δ is small, truth of φ is independent of δ
- It can be computed using a simple extension of the region automaton

Theorem

Robust model-checking of reachability, Büchi, LTL, CoflatMTL properties is decidable. Complexities are those of standard non robust model-checking problems.

[Puri00] Puri. Dynamical properties of timed automata. *Disc. Event Dyn. Syst., 2000.*
[DDMR08] De Wulf, Doyen, Markey, Raskin. Robust safety of timed automata. *FMSD*, 2008.
[BMR06] Bouyer, Markey, Reynier. Robust model-checking of timed automata. *LATIN*, 2006.
[BMR08] Bouyer, Markey, Reynier. Robust analysis of timed automata via channel machines. *FoSSaCS*, 2008.

Outline

Parameterized shrunk semantics for timed automata

A constraint $[a, b]$ **is shrunk to** $[a + k\delta; b - h\delta]$

[SBM11] Sankur, Bouyer, Markey. Shrinking Timed Automata. *FSTTCS*, 2011.

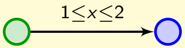
Parameterized shrunk semantics for timed automata

A constraint $[a, b]$ **is shrunk to** $[a + k\delta; b - h\delta]$

Why should we do that?

Abstract model

Real-world model



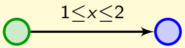
[SBM11] Sankur, Bouyer, Markey. Shrinking Timed Automata. *FSTTCS*, 2011.

Parameterized shrunk semantics for timed automata

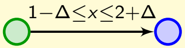
A constraint $[a, b]$ **is shrunk to** $[a + k\delta; b - h\delta]$

Why should we do that?

Abstract model



Real-world model



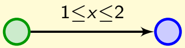
[SBM11] Sankur, Bouyer, Markey. Shrinking Timed Automata. *FSTTCS*, 2011.

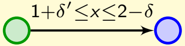
Parameterized shrunk semantics for timed automata

A constraint $[a, b]$ **is shrunk to** $[a + k\delta; b - h\delta]$

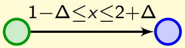
Why should we do that?

Abstract model





Real-world model



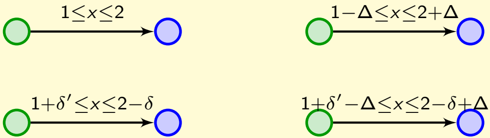
[SBM11] Sankur, Bouyer, Markey. Shrinking Timed Automata. *FSTTCS*, 2011.

Parameterized shrunk semantics for timed automata

A constraint $[a, b]$ **is shrunk to** $[a + k\delta; b - h\delta]$

Why should we do that?

Abstract model | Real-world model



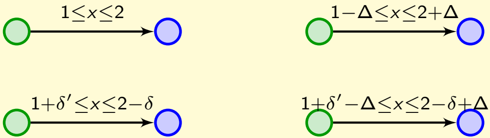
[SBM11] Sankur, Bouyer, Markey. Shrinking Timed Automata. *FSTTCS*, 2011.

Parameterized shrunk semantics for timed automata

A constraint $[a, b]$ is shrunk to $[a + k\delta; b - h\delta]$

Why should we do that?

Abstract model | Real-world model



It is fine as soon as $[1 + \delta' - \Delta; 2 - \delta + \Delta] \subseteq [1; 2]$,
which is the case when $\delta, \delta' \geq \Delta$.

[SBM11] Sankur, Bouyer, Markey. Shrinking Timed Automata. *FSTTCS*, 2011.

Parameterized shrunk semantics for timed automata

A constraint $[a, b]$ **is shrunk to** $[a + k\delta; b - h\delta]$

Summary of the approach

$\leadsto$ Shrink the clock constraints in the model, to prevent additional behaviour in the implementation

- If $\mathcal{B} = \mathcal{A}_{-\mathbf{k}\delta}$, then

$$\mathcal{B} \subseteq \mathtt{program}_\epsilon(\mathcal{B}) \subseteq \mathcal{B}_{f(\epsilon)} = \mathcal{A}_{-\mathbf{k}\delta + f(\epsilon)} \subseteq \mathcal{A}$$

[SBM11] Sankur, Bouyer, Markey. Shrinking Timed Automata. *FSTTCS*, 2011.

Parameterized shrunk semantics – Discussion

What is the relevance of that approach?

Anticipate imprecisions to prevent additional behaviours in the real-world

Parameterized shrunk semantics – Discussion

What is the relevance of that approach?

Anticipate imprecisions to prevent additional behaviours in the real-world

Methodology

- Design and verify $\mathcal{A}$
- Implement $\mathcal{A}_{-\mathbf{k}\delta}$ (parameters are **k** and δ)

Parameterized shrunk semantics – Discussion

What is the relevance of that approach?

Anticipate imprecisions to prevent additional behaviours in the real-world

Methodology

- Design and verify $\mathcal{A}$
- Implement $\mathcal{A}_{-\mathbf{k}\delta}$ (parameters are **k** and δ)

$\rightsquigarrow$ This is good for designing systems with strong/hard timing constraints

Parameterized shrunk semantics – Discussion

What is the relevance of that approach?

Anticipate imprecisions to prevent additional behaviours in the real-world

Methodology

- Design and verify $\mathcal{A}$
- Implement $\mathcal{A}_{-\mathbf{k}\delta}$ (parameters are **k** and δ)

$\rightsquigarrow$ This is good for designing systems with strong/hard timing constraints

⚠ Problem

Make sure that no important behaviours are lost in $\mathcal{A}_{-\mathbf{k}\delta}$!!

Parameterized shrunk semantics – Algorihmics

The (parameterized) shrinkability problem

Find parameters **k** and δ such that:

- $\mathcal{A} \sqsubseteq_{\texttt{t.a.}} \mathcal{A}_{-\mathbf{k}\delta}$ (or $\mathcal{F} \sqsubseteq_{\texttt{t.a.}} \mathcal{A}_{-\mathbf{k}\delta}$ for some finite automaton $\mathcal{F}$) **[shrinkability w.r.t. untimed simulation]**
- $\mathcal{A}_{-\mathbf{k}\delta}$ is non-blocking whenever $\mathcal{A}$ is non-blocking **[shrinkability w.r.t. non-blockingness]**

Parameterized shrunk semantics – Algorihmics

The (parameterized) shrinkability problem

Find parameters **k** and δ such that:

- $\mathcal{A} \sqsubseteq_{\texttt{t.a.}} \mathcal{A}_{-\mathbf{k}\delta}$ (or $\mathcal{F} \sqsubseteq_{\texttt{t.a.}} \mathcal{A}_{-\mathbf{k}\delta}$ for some finite automaton $\mathcal{F}$)
  [shrinkability w.r.t. untimed simulation]
- $\mathcal{A}_{-\mathbf{k}\delta}$ is non-blocking whenever $\mathcal{A}$ is non-blocking
  [shrinkability w.r.t. non-blockingness]

Theorem

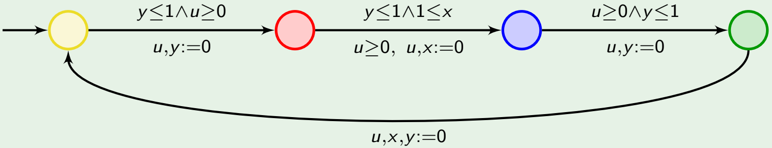
Parameterized shrinkability can be decided (in exponential time).

- Challenge: take care of the accumulation of perturbations
- Technical tools: parameterized shrunk DBM, max-plus equations
- Tool Shrinktech developed by Ocan Sankur [San13]

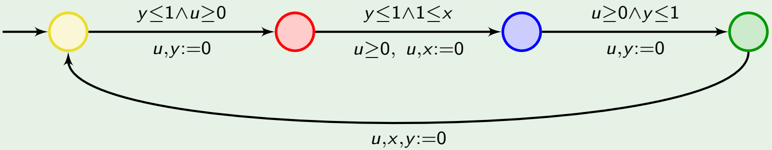
  `http://www.lsv.ens-cachan.fr/Software/shrinktech/`

[San13] Sankur. Shrinktech: A tool for the robustness analysis of timed automata. *CAV*, 2013.

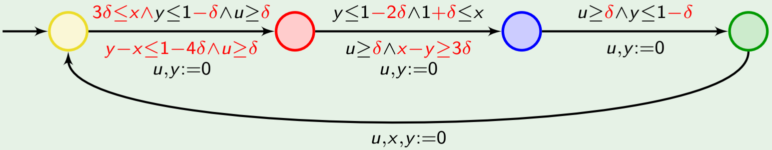
Example



Example



The largest shrunk automaton which is correct w.r.t. untimed simulation and non-blockingness is:



Outline

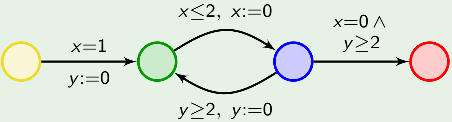
Robust strategy synthesis

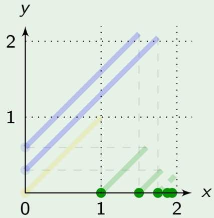
In this talk, a strategy in a timed automaton is a way to resolve (time and action) non-determinism

Robust strategy synthesis

In this talk, a strategy in a timed automaton is a way to resolve (time and action) non-determinism

Example



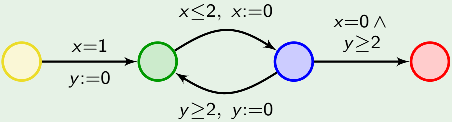


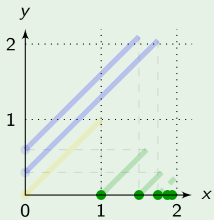
Strategy: in location ◯ with value x, delay $\frac{2-x}{2}$

Robust strategy synthesis

In this talk, a strategy in a timed automaton is a way to resolve (time and action) non-determinism

Example





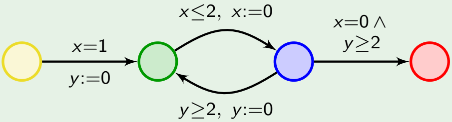
Strategy: in location with value x, delay $\frac{2-x}{2}$

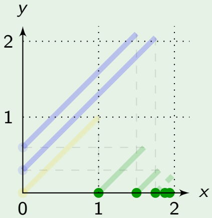
- This strategy requires infinite precision

Robust strategy synthesis

In this talk, a strategy in a timed automaton is a way to resolve (time and action) non-determinism

Example





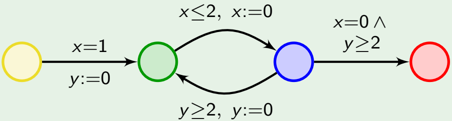
Strategy: in location  with value x, delay $\frac{2-x}{2}$

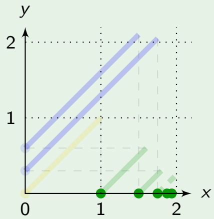
- This strategy requires infinite precision
- In practice, when x is close to 2, no additional delay is supported: the run is theoretically infinite, but it is actually blocking

Robust strategy synthesis

In this talk, a strategy in a timed automaton is a way to resolve (time and action) non-determinism

Example





Strategy: in location  with value x, delay $\frac{2-x}{2}$

- This strategy requires infinite precision
- In practice, when x is close to 2, no additional delay is supported: the run is theoretically infinite, but it is actually blocking
- And that is unavoidable

Robust strategy synthesis

In this talk, a strategy in a timed automaton is a way to resolve (time and action) non-determinism

Idea

Add robustness to strategies, and adapt the behaviour of the system to previous imprecisions

$\rightsquigarrow$ develop a theory of robust strategies that tolerate errors/imprecisions and avoid convergence

Game semantics of a timed automaton

Game semantics $\mathcal{G}_\delta(\mathcal{A})$ of timed automaton $\mathcal{A}$...

... between Controller and Perturbator:

- from (ℓ, v), Controller suggests a delay $d \geq \delta$ and a next edge $e = (\ell \xrightarrow{g,Y} \ell')$ that is available after delay d
- Perturbator then chooses a perturbation $\epsilon \in [-\delta; +\delta]$
- Next state is $(\ell', (v + d + \epsilon)[Y \leftarrow 0])$

Game semantics of a timed automaton

Game semantics $\mathcal{G}_\delta(\mathcal{A})$ of timed automaton $\mathcal{A}$...

... between Controller and Perturbator:

- from (ℓ, v), Controller suggests a delay $d \geq \delta$ and a next edge $e = (\ell \xrightarrow{g,Y} \ell')$ that is available after delay d
- Perturbator then chooses a perturbation $\epsilon \in [-\delta; +\delta]$
- Next state is $(\ell', (v + d + \epsilon)[Y \leftarrow 0])$

Note: when $\delta = 0$, this is the standard semantics of timed automata.

Game semantics of a timed automaton

Game semantics $\mathcal{G}_\delta(\mathcal{A})$ of timed automaton $\mathcal{A}$...

... between Controller and Perturbator:

- from (ℓ, v), Controller suggests a delay $d \geq \delta$ and a next edge $e = (\ell \xrightarrow{g,Y} \ell')$ that is available after delay d
- Perturbator then chooses a perturbation $\epsilon \in [-\delta; +\delta]$
- Next state is $(\ell', (v + d + \epsilon)[Y \leftarrow 0])$

Note: when $\delta = 0$, this is the standard semantics of timed automata.

A δ-robust strategy for Controller is then a strategy that satisfies the expected property, whatever plays Perturbator.

Outline

The excess game semantics

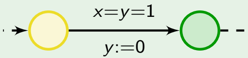
Constraints may not be satisfied after the perturbation: that is, only $v + d$ should satisfy g

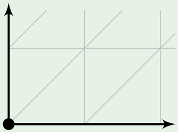
[BMS12] Bouyer, Markey, Sankur. Robust reachability in timed automata: A game-based approach. *ICALP*, 2012.

The excess game semantics

Constraints may not be satisfied after the perturbation: that is, only $v + d$ should satisfy g

Example



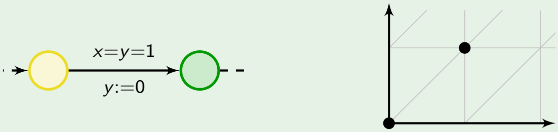


[BMS12] Bouyer, Markey, Sankur. Robust reachability in timed automata: A game-based approach. *ICALP*, 2012.

The excess game semantics

Constraints may not be satisfied after the perturbation: that is, only $v + d$ should satisfy g

Example

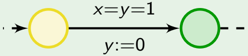


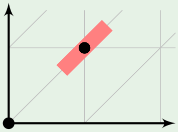
[BMS12] Bouyer, Markey, Sankur. Robust reachability in timed automata: A game-based approach. *ICALP*, 2012.

The excess game semantics

Constraints may not be satisfied after the perturbation: that is, only $v + d$ should satisfy g

Example



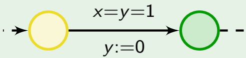


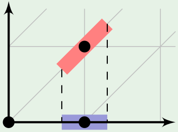
[BMS12] Bouyer, Markey, Sankur. Robust reachability in timed automata: A game-based approach. *ICALP*, 2012.

The excess game semantics

Constraints may not be satisfied after the perturbation: that is, only $v + d$ should satisfy g

Example



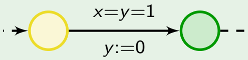


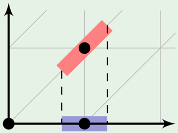
[BMS12] Bouyer, Markey, Sankur. Robust reachability in timed automata: A game-based approach. *ICALP*, 2012.

The excess game semantics

Constraints may not be satisfied after the perturbation: that is, only $v + d$ should satisfy g

Example





$\rightsquigarrow$ Allows simple design of constraints, ensures divergence of time, avoids convergence phenomena

[BMS12] Bouyer, Markey, Sankur. Robust reachability in timed automata: A game-based approach. *ICALP*, 2012.

The excess game semantics – Algorithmics

The (parameterized) synthesis problem

Synthesize $\delta > 0$ and a δ-robust strategy that achieves a given goal.

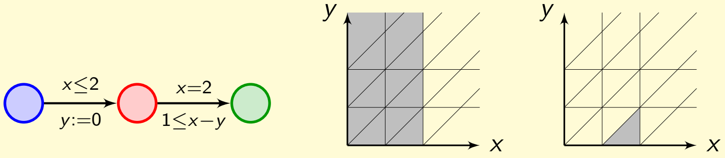
The excess game semantics – Algorithmics

The (parameterized) synthesis problem

Synthesize $\delta > 0$ and a δ-robust strategy that achieves a given goal.

Two challenges

1. Accumulation of perturbations:



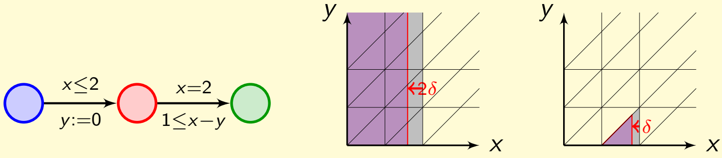
The excess game semantics – Algorithmics

The (parameterized) synthesis problem

Synthesize $\delta > 0$ and a δ-robust strategy that achieves a given goal.

Two challenges

1. Accumulation of perturbations:



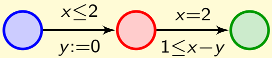
The excess game semantics – Algorithmics

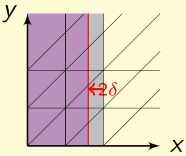
The (parameterized) synthesis problem

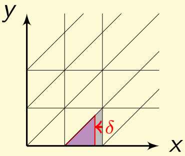
Synthesize $\delta > 0$ and a δ-robust strategy that achieves a given goal.

Two challenges

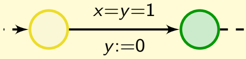
1. Accumulation of perturbations:

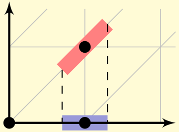






2. New regions become reachable





The excess game semantics – Algorithmics

The (parameterized) synthesis problem

Synthesize $\delta > 0$ and a δ-robust strategy that achieves a given goal.

Theorem

The parameterized synthesis problem for reachability properties is decidable and EXPTIME-complete. Furthermore, uniform winning strategies (w.r.t. δ) can be computed.

- Technical tool: a region-based refined game abstraction
- ☺ Extends to two-player games (i.e. to real control problems)
- ☹ Only valid for reachability properties

Outline

The conservative game semantics

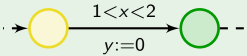
Constraints have to be satisfied after the perturbation: that is, $v + d + \epsilon$ should satisfy g for every $\epsilon \in [-\delta; +\delta]$

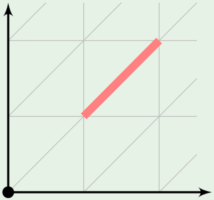
[SBMR13] Sankur, Bouyer, Markey, Reynier. Robust Controller Synthesis in Timed Automata. *Under submission.*

The conservative game semantics

Constraints have to be satisfied after the perturbation: that is, $v + d + \epsilon$ should satisfy g for every $\epsilon \in [-\delta; +\delta]$

Example



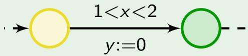


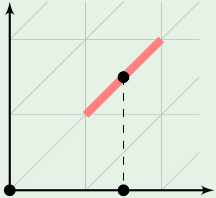
[SBMR13] Sankur, Bouyer, Markey, Reynier. Robust Controller Synthesis in Timed Automata. *Under submission.*

The conservative game semantics

Constraints have to be satisfied after the perturbation: that is, $v + d + \epsilon$ should satisfy g for every $\epsilon \in [-\delta; +\delta]$

Example





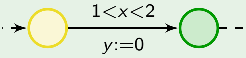
$\rightsquigarrow$ Strongly ensures timing constraints, ensures divergence of time, prevents converging phenomena

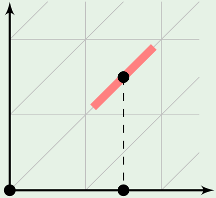
[SBMR13] Sankur, Bouyer, Markey, Reynier. Robust Controller Synthesis in Timed Automata. *Under submission.*

The conservative game semantics

Constraints have to be satisfied after the perturbation: that is, $v + d + \epsilon$ should satisfy g for every $\epsilon \in [-\delta; +\delta]$

Example





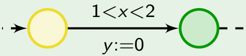
$\rightsquigarrow$ Strongly ensures timing constraints, ensures divergence of time, prevents converging phenomena

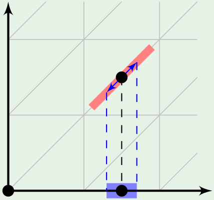
[SBMR13] Sankur, Bouyer, Markey, Reynier. Robust Controller Synthesis in Timed Automata. *Under submission.*

The conservative game semantics

Constraints have to be satisfied after the perturbation: that is, $v + d + \epsilon$ should satisfy g for every $\epsilon \in [-\delta; +\delta]$

Example





$\rightsquigarrow$ Strongly ensures timing constraints, ensures divergence of time, prevents converging phenomena

[SBMR13] Sankur, Bouyer, Markey, Reynier. Robust Controller Synthesis in Timed Automata. *Under submission*.

The conservative game semantics – Algorithmics

The (parameterized) synthesis problem

Synthesize $\delta > 0$ and a δ-robust strategy that achieves a given goal.

The conservative game semantics – Algorithmics

The (parameterized) synthesis problem

Synthesize $\delta > 0$ and a δ-robust strategy that achieves a given goal.

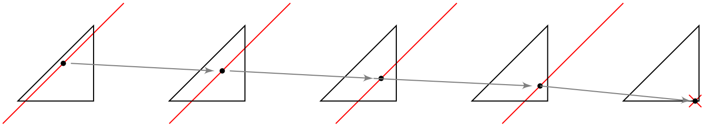
Theorem

The synthesis problem for Büchi properties is decidable and PSPACE-complete. Furthermore, δ is at most doubly-exponential, and uniform winning strategies (w.r.t. δ) can be computed.

The problem consists in finding cycles that do not become blocked.

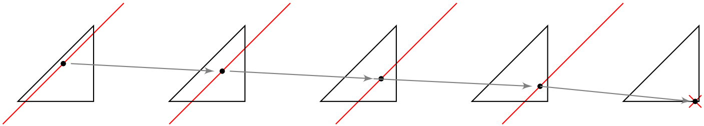
The problem consists in finding cycles that do not become blocked.

- A converging phenomena:

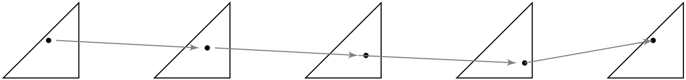


The problem consists in finding cycles that do not become blocked.

- A converging phenomena:



- No convergence:

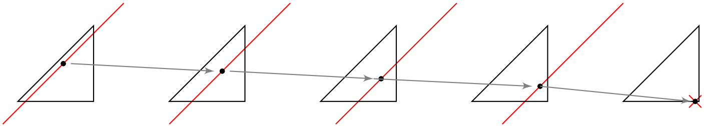


No such constraining half-spaces.

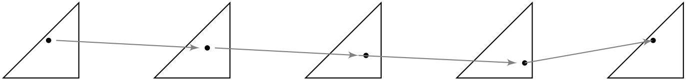
[AB11] Asarin, Basset. Thin and Thick Timed Regular Languages. *FORMATS*, 2011.

The problem consists in finding cycles that do not become blocked.

- A converging phenomena:



- No convergence:



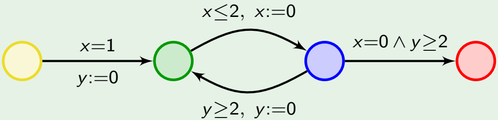
No such constraining half-spaces.

Tools for solving the synthesis problem

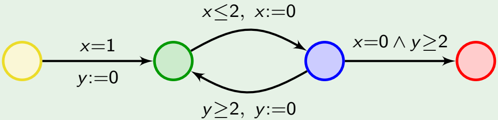
- Orbit graphs, forgetful cycles [AB11]
- Forgetful (that is, strongly connected) orbit graph $\Leftrightarrow$ no convergence phenomena
  $\leadsto$ strong relation with thick automata.

[AB11] Asarin, Basset. Thin and Thick Timed Regular Languages. *FORMATS*, 2011.

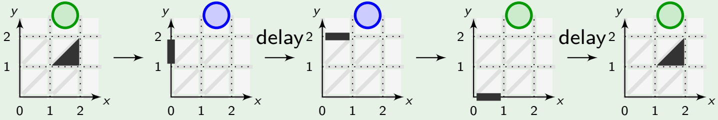
Example



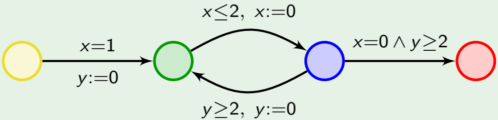
Example



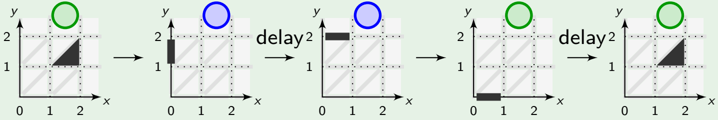
A region cycle:



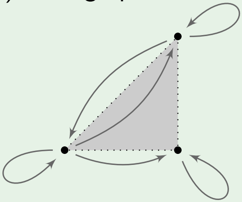
Example



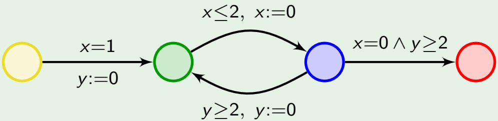
A region cycle:



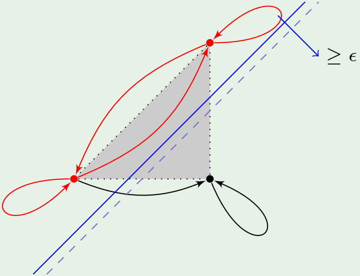
The corresponding (folded) orbit graph:



Example



The cycle is not forgetful (that is, not strongly connected), Perturbator can enforce convergence:



Outline

Conclusion

- Timed automata: a nice mathematical model for real-time systems with interesting decidability properties and algorithmics solutions.
- Not always easy to transfer correctness proven in this model to real behaviours of the system.
- We have shown several frameworks for robustness that can be used to ensure correctness in the real-world..

Conclusion

- Timed automata: a nice mathematical model for real-time systems with interesting decidability properties and algorithmics solutions.
- Not always easy to transfer correctness proven in this model to real behaviours of the system.
- We have shown several frameworks for robustness that can be used to ensure correctness in the real-world..

- Extension of these works to richer models seems unfortunately hard [BMS13]
- A quantitative approach to robustness: Perturbator plays randomly
- Symbolic algorithms?

Conclusion

- Timed automata: a nice mathematical model for real-time systems with interesting decidability properties and algorithmics solutions.
- Not always easy to transfer correctness proven in this model to real behaviours of the system.
- We have shown several frameworks for robustness that can be used to ensure correctness in the real-world..

- Extension of these works to richer models seems unfortunately hard [BMS13]
- A quantitative approach to robustness: Perturbator plays randomly
- Symbolic algorithms?

- This list of possible approaches is not exhaustive:
  - tube acceptance [GHJ97]
  - turn any automaton into a robust one [BLM+11]
  - sampling approach [KP05,BLM+11]
  - probabilistic approach [BBB+08,BBJM12]
  - ...