Networked Robust Stabilization: when gap meets two-port

Guoxiang Gu and Li Qiu

Louisiana State University
Hong Kong University of Science and Technology

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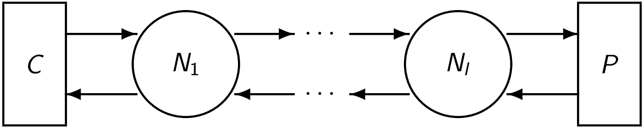
Networked control system (NCS)



- The plant and controller are uncertain and the uncertainty is described by the gap metric.
- The communication network is a cascade of two-port networks, modeling bidirectional transmission with relays.
- We only consider SISO plants and controllers in this talk, for the sake of simplicity.

Gap metric: a review

(Zames; Vidyasagar; Georgiou and Smith; Qiu and Davison; Vinnicombe, from 1980 to ∼1995.)

- P is LTI and possibly unstable.
- Graph of P

$$\mathcal{G}_P = \left\{ \begin{bmatrix} u \\ y \end{bmatrix} \in \mathcal{H}_2 \times \mathcal{H}_2 : y = Pu \right\}.$$

  a subspace of $\mathcal{H}_2 \times \mathcal{H}_2$.

- Gap metric between P_1 and P_2.

$$\delta(P_1, P_2) = \|\Pi_{\mathcal{G}_{P_1}} - \Pi_{\mathcal{G}_{P_2}}\|.$$

- Uncertain systems can be described by gap balls

$$\mathcal{B}(P, r) = \{\tilde{P} : \delta(P, \tilde{P}) \leq r\}.$$

- Gap ball viewed as rotation of the graph:

$$\{\tilde{P} : \mathcal{G}_{\tilde{P}} = (I + \Delta)\mathcal{G}_P, \|\Delta\|_\infty \leq r\} \subset \mathcal{B}(P, r).$$

  The maximal rotating angle is $\arcsin r$.

Gap metric: a review (continued)

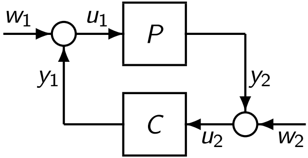


Figure: Feedback System (P, C).

- The gang of four

$$GoF(P, C) = \begin{bmatrix} \frac{1}{1-PC} & \frac{-C}{1-PC} \\ \frac{P}{1-PC} & \frac{-PC}{1-PC} \end{bmatrix} = \begin{bmatrix} 1 \\ P \end{bmatrix} (1 - PC)^{-1} \begin{bmatrix} 1 & -C \end{bmatrix}.$$

- Need to work on the inverse graph of C

$$\mathcal{G}'_C = \left\{ \begin{bmatrix} u \\ y \end{bmatrix} \in \mathcal{H}_2 \times \mathcal{H}_2 : u = Cy \right\}.$$

- (P, C) is stable if $\mathcal{G}_P$ and $\mathcal{G}'_C$ are complementary.

Gap metric: a review (continued)

- $(\tilde{P}, \tilde{C})$ is stable for all $\tilde{P} \in \mathcal{B}(P, r_P)$ and $\tilde{C} \in \mathcal{B}(C, r_C)$ iff

$$\arcsin r_P + \arcsin r_C < \arcsin \|GoF(P, C)\|_\infty^{-1}.$$

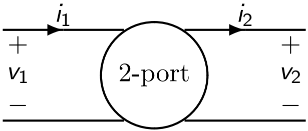
  (arcsin theorem)

- Optimal robust control problem:

$$\min_C \|GoF(P, C)\|_\infty$$

  an "easy" $\mathcal{H}_\infty$ control problem.

Two-port circuit: a review



- Transmission representation

$$\begin{bmatrix} v_1(s) \\ i_1(s) \end{bmatrix} = A(s) \begin{bmatrix} v_2(s) \\ i_2(s) \end{bmatrix}.$$

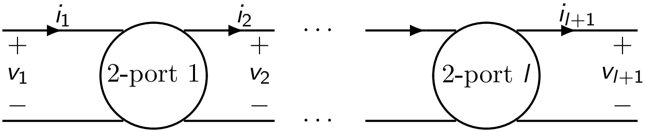
- Ideal transmission

$$A_0(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

- Transmission with distortion

$$A(s) = I + \Delta(s).$$

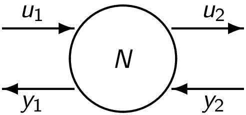
Cascade connection of two-port circuits



- The transmission matrices are multiplied

$$\begin{bmatrix} v_1(s) \\ i_1(s) \end{bmatrix} = A_1(s)A_2(s)...A_l(s)\begin{bmatrix} v_{l+1}(s) \\ i_{l+1}(s) \end{bmatrix}.$$

Two-port communication network



- Voltages $\rightarrow$ down-link signals, currents $\rightarrow$ up-link signals.
- Transmission matrix

$$\begin{bmatrix} u_1 \\ y_1 \end{bmatrix} = A \begin{bmatrix} u_2 \\ y_2 \end{bmatrix}.$$

- Ideal transmission

$$A_0 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

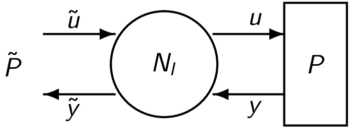
- Transmission with distortion

$$A = I + \Delta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \Delta_{\div} & \Delta_{-} \\ \Delta_{+} & \Delta_{\times} \end{bmatrix}, \quad \|\Delta\|_\infty < r.$$

(Notation invented by Halsey and Glover)

- We allow Δ to be nonlinear.

Plant with two-port distortion



- Linear fractional transformation (LFT)

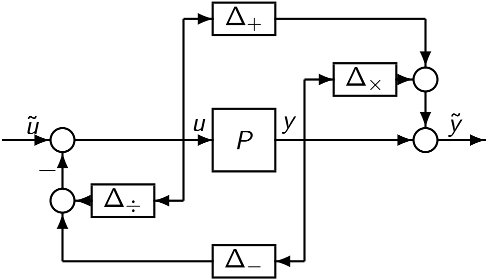
$$\tilde{P} = \frac{(1+\Delta_\times)P + \Delta_+}{1+\Delta_\div + \Delta_- P}.$$

- Graph of the distorted system

$$\mathcal{G}_{\tilde{P}} = (I+\Delta)\mathcal{G}_P.$$

The same type of rotation as in the gap ball.

System with $+, -, \times, \div$ uncertainty.



Two-port transmission model of NCS

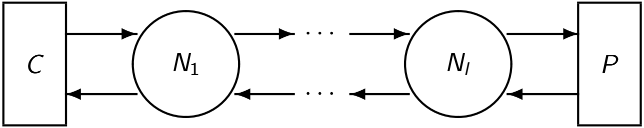


Figure: Networked control system.

- $\tilde{P}, \tilde{C}$ are only known to belong to gap balls

$$\tilde{P} \in \mathcal{B}(P, r_P), \tilde{C} \in \mathcal{B}(C, r_C).$$

- N_i is only known to have transmission matrix

$$A_i = I + \Delta_i, \|\Delta_i\|_\infty \leq r_i.$$

Main result

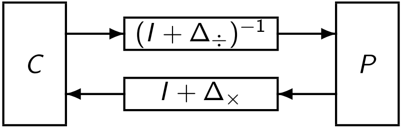
- The NCS is robustly stable iff

$$\arcsin r_P + \arcsin r_C + \sum_{i=1}^{l} \arcsin r_i < \arcsin \|GoF(P, C)\|_\infty^{-1}.$$

  (Networked arcsin theorem)
- Optimal design

$$\min_C \|GoF(P, C)\|_\infty.$$

Hammerstein-Wiener model of NCS



- $\Delta_{\times}$ and $\Delta_{\div}$ are nonlinear time-varying systems satisfying

$$\|\Delta_{\times}\|_{\infty} \leq r, \quad \|\Delta_{\div}\|_{\infty} \leq r.$$

- Two-port network with diagonal transmission matrix.
- $\Delta_{\times}$ and $\Delta_{\div}$ can be used to model logarithmic quantizations.
- The NCS is robustly stable iff

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \Delta_{\div} & 0 \\ 0 & \Delta_{\times} \end{bmatrix} GoF$$

is stably invertible for all $\Delta_{\times}$ and $\Delta_{\div}$.

μ-synthesis of Hammerstein-Wiener system

- Introducing scaling: the closed-loop system is robustly stable iff

$$\begin{aligned}
&\inf_{\gamma\in(0,\infty)} \left\| \begin{bmatrix} 1 & 0 \\ 0 & \gamma \end{bmatrix} GoF(P,C) \begin{bmatrix} 1 & 0 \\ 0 & \gamma^{-1} \end{bmatrix} \right\|_\infty \\
= &\inf_{\gamma\in(0,\infty)} \left\| \begin{bmatrix} \frac{1}{1-PC} & \frac{-\gamma^{-1}C}{1-PC} \\ \frac{\gamma P}{1-PC} & \frac{PC}{1-PC} \end{bmatrix} \right\|_\infty \\
= &\inf_{\gamma\in(0,\infty)} \|GoF(\gamma P, \gamma^{-1}C)\|_\infty < \frac{1}{r}.
\end{aligned}$$

- Optimal design

$$\inf_{\gamma\in(0,\infty)} \inf_C \|GoF(\gamma P, \gamma^{-1}C)\|_\infty.$$

- The function $\gamma \mapsto \inf_C \|GoF(\gamma P, \gamma^{-1}C)\|_\infty$ was mistakenly conjectured to be unimodal.

Conclusions

- Trade-off between the capacities of the down-link and the up-link channels.
- Optimal robust networked control, linking the history.
- H_2 vs H_∞ theory