STRUCTURE AND STABILITY IN FEEDBACK NETWORKS

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OUTLINE

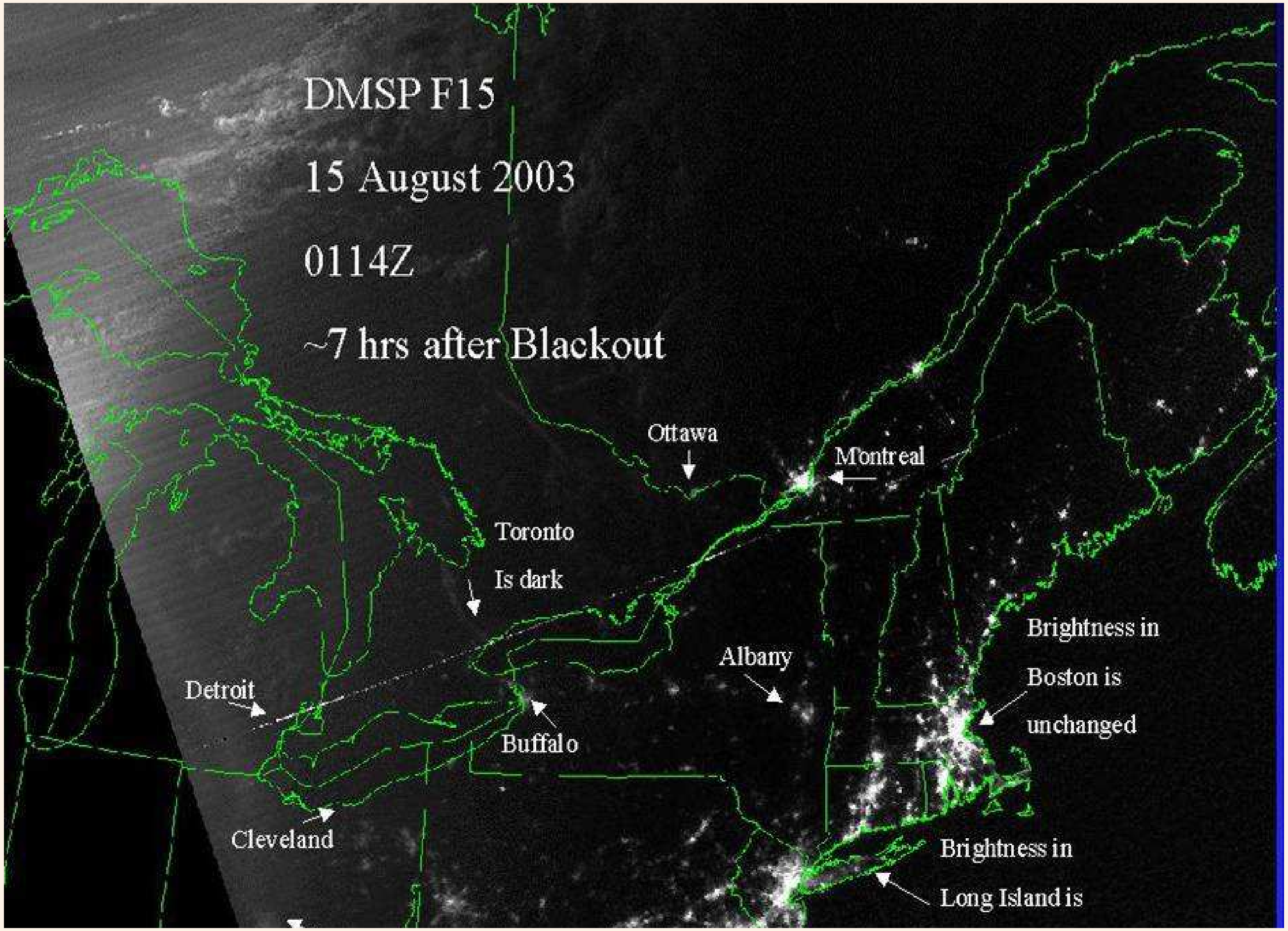
- FEEDBACK NETWORKS
- SYNCHRONIZATION
- CONTROL DESIGN
- OTHER WORK
- CONCLUSIONS

FEEDBACK NETWORKS

- Motivation
- Complexity
- Graphs of control
- Laplacians

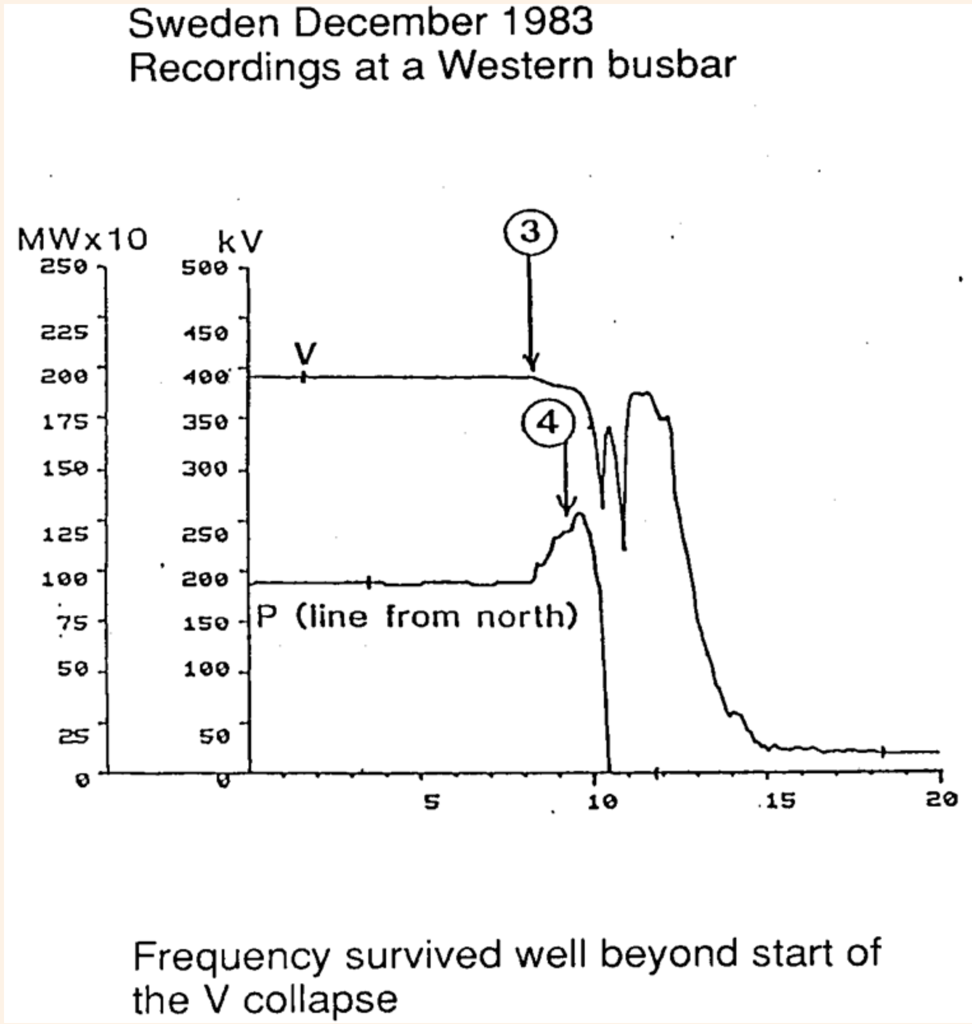
FEEDBACK NETWORKS

Blackout 2003 USA-Canada



FEEDBACK NETWORKS

Voltage Collapse



FEEDBACK NETWORKS

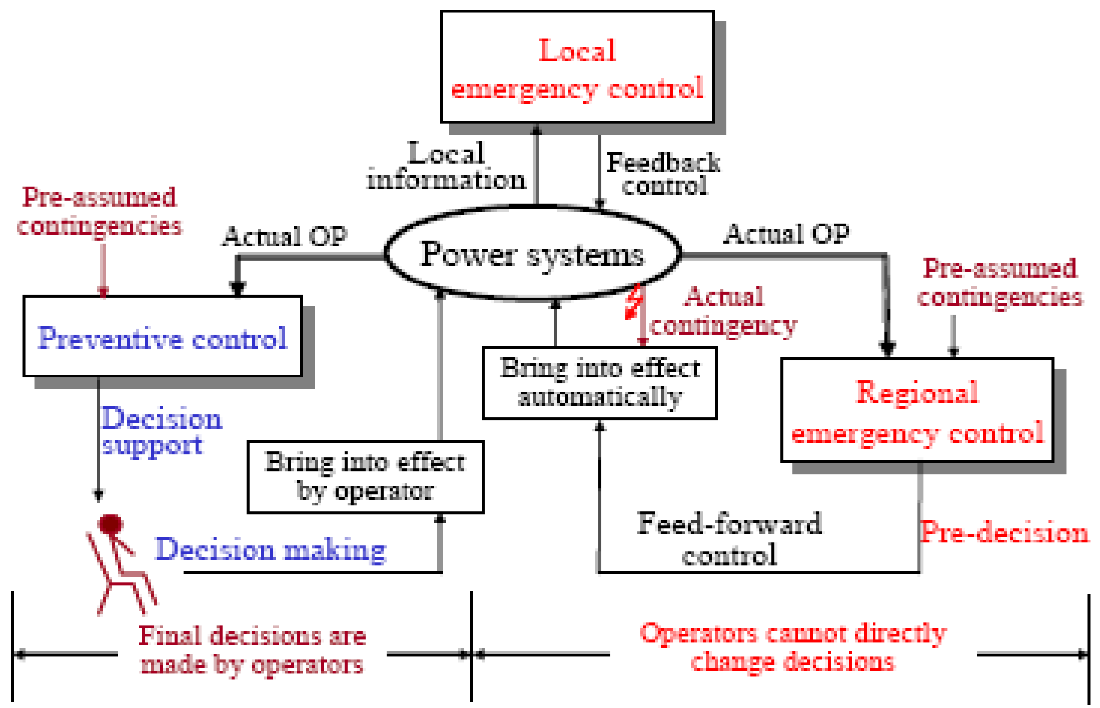


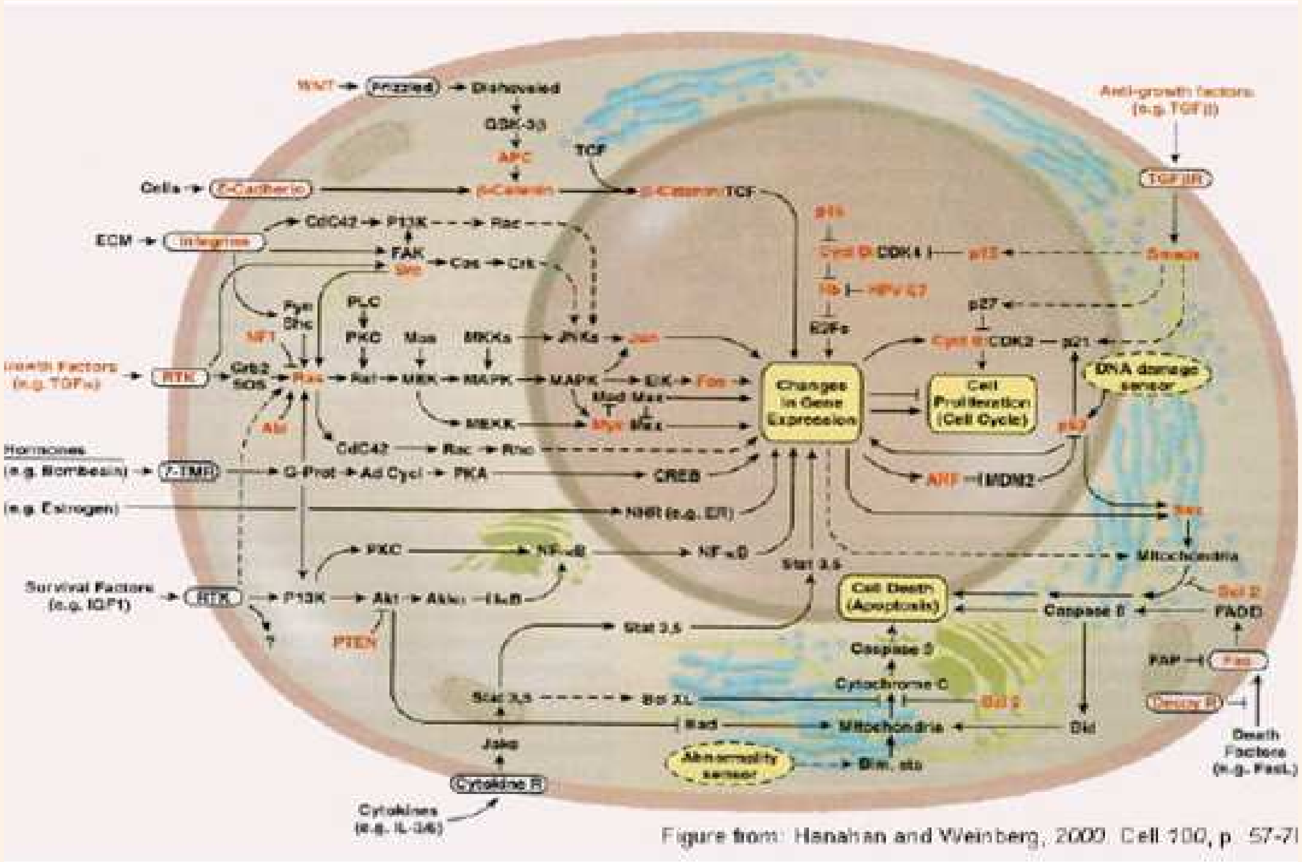
Figure 1: Three kinds of stability controls

Thousands of distributed control actions arranged in hierarchy.

Ref: Yusheng Xue, PSCC 2005

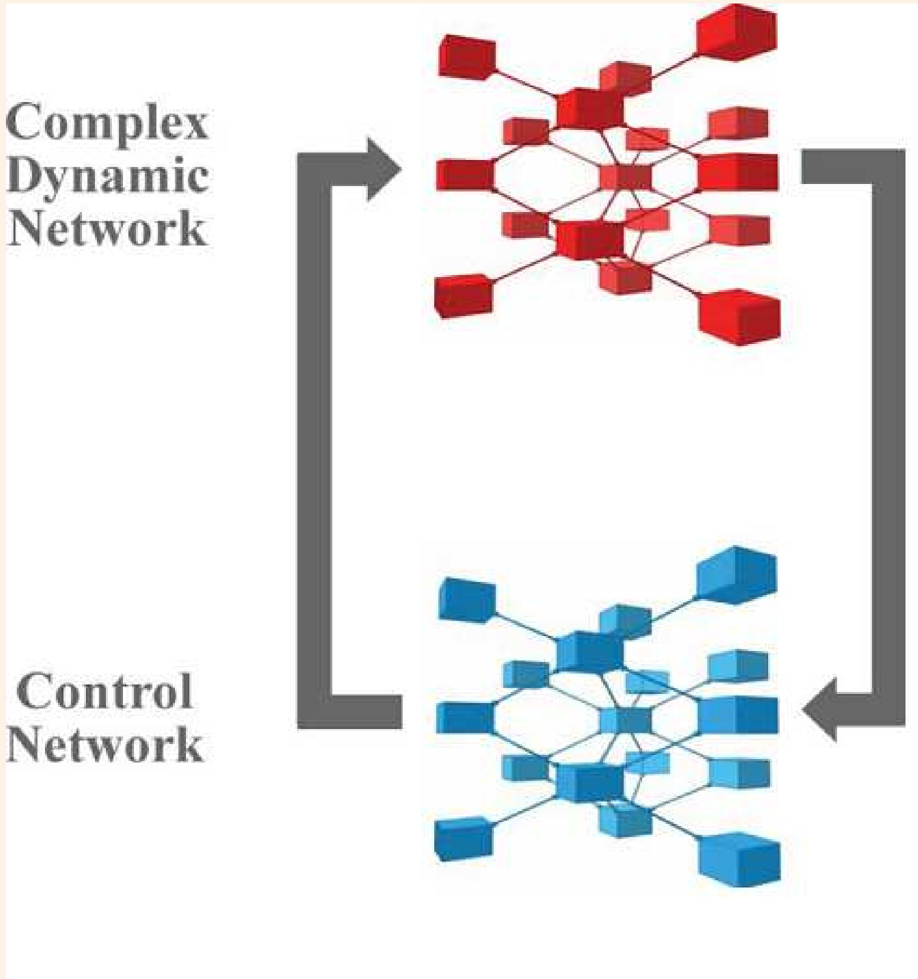
FEEDBACK NETWORKS

Network of Life



FEEDBACK NETWORKS

Network Control of a Network



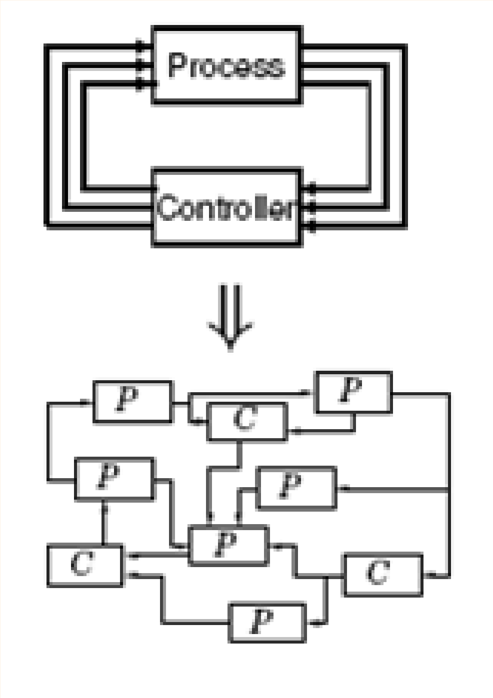
Networks time-varying, switched, nonlinear

FEEDBACK NETWORKS

Special cases

- Decentralized control
- Distributed control

Key difference: now ask questions about architecture, switching algorithms, etc



Ref: Rantzer, CDC, 2008

FEEDBACK NETWORKS

Graphs of Control

The system is a large network (system graph)

– Cannot be controlled centrally

Controllers will need to communicate (control graph)

Sensing of data (sensor graph)

– Control designed around multiple graphs

FEEDBACK NETWORKS

General Network Model

We consider the general dynamic network consisting of:

– diffusive coupling;

– massive numbers of nodes modelled as n-dimensional systems

$$\dot{x}_i = f_i(x_i, p_i) + \sum_{j=1, j\neq i}^{N} a_{ij}\Gamma(x_j - x_i) + G_i u_i, \quad i = 1, \ldots, N;$$

Special case (Networks science): network with uniform coupling and linearly interconnected identical nodes

$$\dot{x}_i = f(x_i) + c \sum_{j=1, j\neq i}^{N} a_{ij}\Gamma(x_j - x_i) + u_i, \quad i = 1, \ldots, N,$$

or

$$\dot{x}_i = f(x_i) + c \sum_{j=1}^{N} a_{ij}\Gamma x_j + u_i, \quad i = 1, \ldots, N.$$

FEEDBACK NETWORKS

Laplacian Matrix

Outer coupling matrix A represents the topology of the network

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix}$$

where $a_{ij} > 0$, if there is a connection between nodes i and j, $a_{ij} = 0$, otherwise, and

$$a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$$

Laplacian $L = -A$

FEEDBACK NETWORKS

Properties of L

Consider unweighted connected case.

Eigenvalues: $0 = \lambda_1 < \lambda_2 \leq \lambda_3 \leq \cdots \leq \lambda_N$

Properties:

- $\lambda_2 = 0$ if graph disconnected
- $\lambda_2 \leq N/(N-1)$. min deg(k)
- $\lambda_2 \geq 4/N$. Diameter
- $\lambda_N \geq N/(N-1)$. max deg(k)
- $\lambda_N \leq N$.

Ref: Chung, Spectral Graph Theory, AMS. 1997.

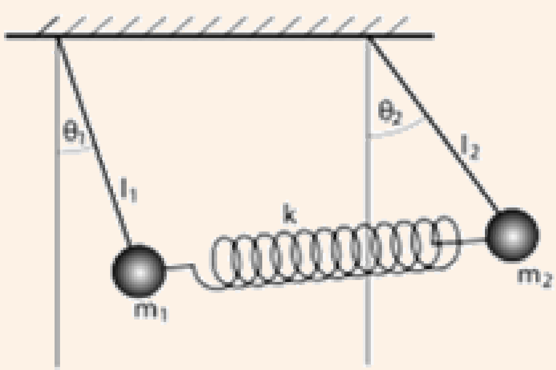
FEEDBACK NETWORKS

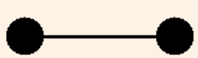
Pendulum Model

Coupled Pendulums are modeled by

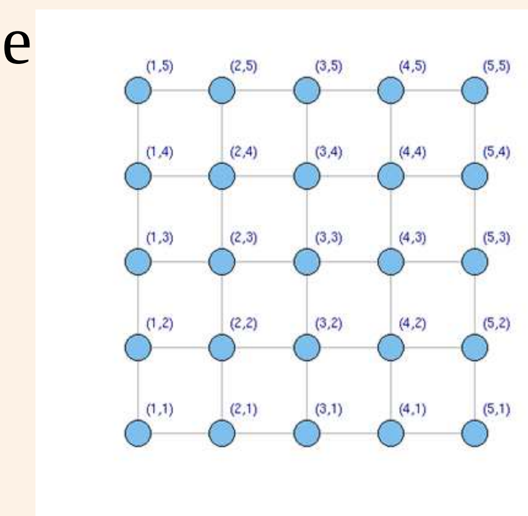
$$m_i\ddot{\theta}_i + \gamma_i\dot{\theta}_i + b_i \sin\theta_i = \tau_i' + \tau_i \sin(\omega t + \phi_i) + \sum_{j=1, j\neq i}^{N} b_{ij}(\theta_j - \theta_i)$$

2 coupled pendulums represented by a 2 node network with 1 link





Coupled pendulums can be arranged in network structure such as a 2-D Lattice



FEEDBACK NETWORKS

Control by a Network

Controllers with no communication time-delays

$$u_i = \sum_{j=1, j\neq i}^{N} b_{ij}\Lambda(x_j - x_i), i = 1, \cdots, N; \qquad (1)$$

B is the coupling matrix of the controllers (1) (has the same properties as A), which gives a Laplacian $(-B)$ for the controllers.

Special case (R. Olfati-Saber & R.M. Murray IEEE AC 2004)

$$\dot{x}_i = u_i$$

with u_i having the form of (1).

FEEDBACK NETWORKS

Meta-view

- No control network (Laplacian -A)
  - Sync results (Pecora and Carroll, 1998; Wang and Chen, 2002)
  - Vulnerability and fragility (Wang and Chen, 2002; Doyle, et al., 2005)
  - Identical nodes model
- No system network (Laplacian -B)
  - Consensus results (Olfati-Saber and Murray, 2004; Su and Wang, 2009; etc)
  - Identical agents
  - Switching, time-delays

FEEDBACK NETWORKS

Feedback Networks

- Both network system and network control
- Scale on connectivity and dynamics
- Nonlinearity, switching, time-delays
- Structure important to performance and security, i.e. system planning, control architecture
- Heaps of stability theory on interconnected systems; some uses structure explicitly, but not much is directly useful here, e.g. multiple equilibria, oscillations etc
- Some ideas in power system theory are useful
- Stability theory in network science very simple, i.e. local, but does use the graph

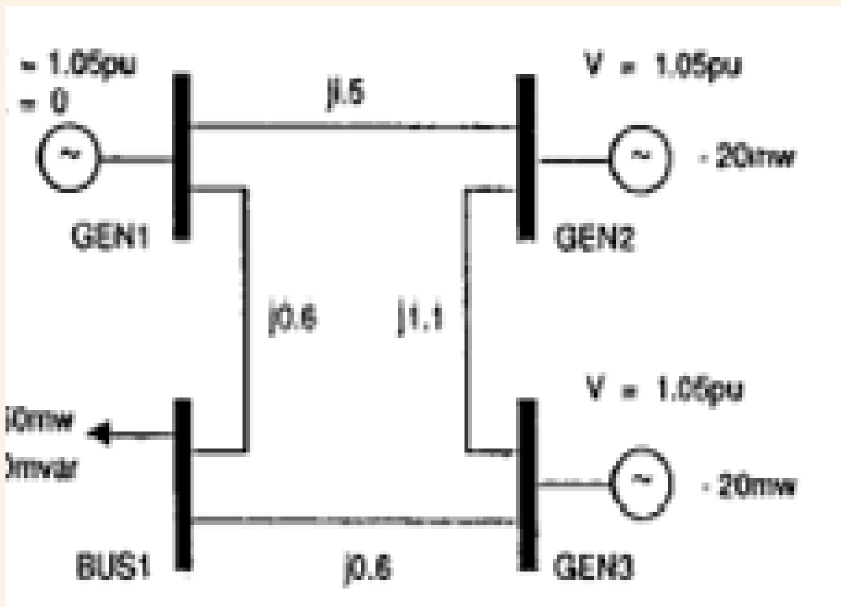
SYNCHRONIZATION

- More on power systems
- Review complex networks with identical nodes
- Bounded sync with non-id nodes
- Asymptotic sync with non-id nodes

SYNCHRONIZATION

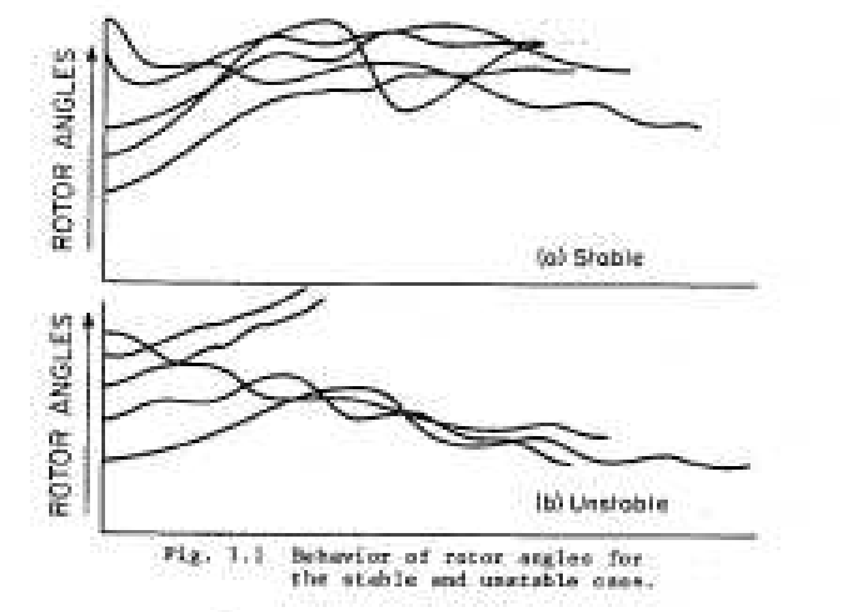
Power systems as dynamic network

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i + \sum_{j\in C_i} V_iV_jb_{ij}\sin(\theta_i - \theta_j) = P_i$$



SYNCHRONIZATION

Phase Angle Stability in Power Networks



Ref: M.A.Pai, Energy Function Analysis for Power System Stability

Review complex networks with identical nodes

Network model:

$$\dot{x}_i = f(x_i) + c \sum_{j=1}^{N} a_{ij} \Gamma x_j, \quad i = 1, \ldots, N,$$

$x_i = (x_{i1}, \ldots, x_{in})^T \in R^n$: state of the i-th node
$x = (x_1^T, \ldots, x_N^T)^T \in R^{nN}$: state of the network
$A = (a_{ij})_{N \times N}$: outer coupling matrix,

- symmetric
- $\sum_{j=1}^{N} a_{ij} = 0$, $i = 1, \ldots, N$

Γ : inner coupling matrix,
f : continuously differentiable with Jacobian Df.

SYNCHRONIZATION

Review complex networks with identical nodes

- Synchronization: $x_i(t) - x_j(t) \to 0, i, j = 1, \cdots, N$
- Synchronization manifold: $\{x \mid x_1 = x_2 = \cdots = x_N\}$
- Remarks:
  - A network can be regarded as a dynamical system, synchronization can be viewed as some type of stability issue (not usual one)
  - Large number of nodes $\Rightarrow$huge dimension
  - Synchronization criteria need to be checkable, computable, usually of lower dimension
  - Identical nodes $\Rightarrow$ invariant synchronization manifold

SYNCHRONIZATION

Review complex networks with identical nodes

Consider solution for an isolated node: $s(t)$

$$\dot{s}(t) = f(s(t))$$

Let unitary matrix $\Phi = (\varphi_{ij})_{N\times N} = (\Phi_1, \ldots, \Phi_N)$,

$$\Phi^T A \Phi = \Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_N\},$$

Errors: $e_i = x_i - s(t)$, $e = (e_1^T, \cdots, e_N^T)^T$

$$\dot{e}_i \;=\; f(e_i + s) - f(s(t)) + c\textstyle\sum_{j=1}^N a_{ij}\Gamma e_j$$

$$\text{linearized} \;\rightarrow\; Df(s)e_i + c\textstyle\sum_{j=1}^N a_{ij}\Gamma e_j$$

SYNCHRONIZATION

Review complex networks with identical nodes

or,

$$\dot{e} = (I \otimes Df + cA \otimes \Gamma)e$$

Let $\omega = (\Phi^T \otimes I_n)e$,

$$\dot{\omega} = (I \otimes Df + c\Lambda \otimes \Gamma)\omega$$

i.e. $$\dot{\omega}_i = (Df + c\lambda_i \Gamma)\omega_i, \quad i = 1, 2, \cdots, N \qquad (2)$$

Theorem: Local synchronization $\Leftrightarrow$ Simultaneous asymptotic stability of (2)

One sufficiency criterion: $c \geq \dfrac{|\bar{d}|}{|\lambda_2|}$

(Wang and Chen, 2002)

SYNCHRONIZATION

Review complex networks with identical nodes

Remarks

- Many extensions to include time delay, uncertainties, switching topology...
- Some extensions to nonlinear outer coupling
- Some global versions: robustness analysis

Bounded synchronization

Network model:

$$\dot{x}_i = f_i(x_i) + c \sum_{j=1}^{N} a_{ij} \Gamma x_j, \quad i = 1, \ldots, N, \qquad (3)$$

- In many cases, asymptotic synchronization $e \longrightarrow 0$ is impossible mainly because of non-identical nodes.

- How to describe the synchronization behavior?

  Boundedness! $\quad e \longrightarrow$ some set

- Have a precise bound

SYNCHRONIZATION

Bounded sync with non-id nodes

Define: $e_i = x_i - s(t)$, $e = (e_1^T, \ldots, e_N^T)^T$.

$$\dot{e}_i \;=\; f_i(s) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j + \int_0^1 Df_i(s+\tau e_i)e_i d\tau - \dot{s}. \qquad (4)$$

$$\begin{aligned}
&\dot{e} = (cA \otimes \Gamma)e \\
&+\mathrm{diag}\left\{\int_0^1 Df_1(s+\tau e_1)d\tau, \cdots, \int_0^1 Df_N(s+\tau e_N)d\tau\right\}e \qquad (5)\\
&+\left(f_1^T(s), \cdots, f_N^T(s)\right)^T - \left(\dot{s}^T, \cdots, \dot{s}^T\right)^T.
\end{aligned}$$

Remark: Unified form of error equation

SYNCHRONIZATION

Bounded sync with non-id nodes

s is the average trajectory

- Average state trajectory $s(t) = \frac{1}{N}\sum_{k=1}^{N} x_k(t)$
- Average dynamics $\bar{f}(x) = \frac{1}{N}\sum_{k=1}^{N} f_k(x)$
- Obviously, $\sum_{i=1}^{N} e_i = 0$

SYNCHRONIZATION

Bounded sync with non-id nodes

$$\dot{e} = (cA \otimes \Gamma)e + \text{diag}\{\int_0^1 (Df_1(s+\tau e_1)d\tau \cdots \int_0^1 (Df_N(s+\tau e_N)d\tau\}e + \begin{pmatrix} f_1(s) - \bar{f}(s) \\ \vdots \\ f_N(s) - \bar{f}(s) \end{pmatrix}. \qquad (6)$$

$$-\frac{1}{N}\begin{pmatrix} \int_0^1 Df_1(s+\tau e_1)d\tau & \cdots & \int_0^1 Df_N(s+\tau e_N)d\tau \\ \vdots & \ddots & \vdots \\ \int_0^1 Df_1(s+\tau e_1)d\tau & \cdots & \int_0^1 Df_N(s+\tau e_N)d\tau \end{pmatrix} e$$

- $e = 0$ is no longer an equilibrium point
- attractiveness to the origin $\Rightarrow$ synchronization

SYNCHRONIZATION

Bounded sync with non-id nodes

Let $\mathscr{PC}^1_{n\times n}$ be the linear space of the uniformly bounded continuously differentiable real $n \times n$ matrix-valued functions defined on $[0, \infty)$.

Theorem. Suppose there exist uniformly positive definite matrices $P_i(t) \in \mathscr{PC}^1_{n\times n}$, constant $a > 0, b > 0$, functions $\alpha(t) > 0$ and $\gamma(t) \geq 0$ such that

$$a\|x\|^2 \leq x^T P_i(t) x \leq b\|x\|^2, \quad \forall t \in R_+,\ x \in R^n,\ i = 2, \ldots, N, \tag{7}$$

$$\dot{P}_i + P_i(D\bar{f}(s) + c\lambda_i\Gamma) + (D\bar{f}(s) + c\lambda_i\Gamma)^T P_i + \alpha(t) I \prec 0, \\ i = 2, \ldots, N, \tag{8}$$

$$\left\| \int_0^1 (Df_i(s + \tau e_i) - D\bar{f}(s)) d\tau \right\| \leq \gamma, i = 1, \ldots, N. \tag{9}$$

SYNCHRONIZATION

Bounded sync with non-id nodes

Let

$$\mu(t) = \left\| \begin{pmatrix} f_1(s) - \bar{f}(s) \\ \vdots \\ f_N(s) - \bar{f}(s) \end{pmatrix} \right\|, \qquad (10)$$

$$\beta = (\sum_{i=2}^{N} \|P_i\|^2)^{\frac{1}{2}}. \qquad (11)$$

If $\alpha(t) - 2\gamma(t)\beta \geq \bar{\delta}$ for some constant $\bar{\delta} > 0$, the error $e(t)$ converges to the set

$$\bar{Q} = \left\{ e \middle| \|e\| \leq 2\beta \sqrt{\frac{b}{a}} \, \overline{\lim}_{t\to\infty} \frac{\mu(t)}{\alpha(t) - 2\gamma(t)\beta} \right\}. \qquad (12)$$

SYNCHRONIZATION

Bounded sync with non-id nodes

Corollary. When $\overline{\lim}_{t\to\infty}\mu(t) = 0$, we have asymptotic synchronization in the classical sense. In particular, when $f_i = f$, that is, all nodes are identical, we have $\mu(t) \equiv 0$.

Asymptotic synchronization

Proposition. Suppose

- $x_i(t)$ are uniformly continuous with respect to t,
- $f_i(x)$ are uniformly continuous with respect to x.

If the network (3) synchronizes, then,

$$\lim_{t\to\infty}(f_i(s(t)) - f_j(s(t))) = 0, 1 \leq i, j \leq N \tag{13}$$

SYNCHRONIZATION

Asymptotic sync with non-id nodes

Theorem. Suppose

(i) $$\lim_{t\to\infty}(f_i(s(t)) - f_j(s(t))) = 0, 1 \le i, j \le N,$$

(ii) there exist time-varying matrix Π, uniformly positive definite matrices $P_i(t) \in \mathscr{PC}^1_{n\times n}$ with $\| P_i \| \le 1$ and constant $\alpha > 0$ such that

$$\dot{P}_i(t) + P_i(t)(\Pi + c\lambda_i\Gamma) + (\Pi + c\lambda_i\Gamma)^T P_i(t) + \alpha I \prec 0, \quad i = 2, \ldots, N, \tag{14}$$

(iii) $$\| \int_0^1 Df_i(s + \tau e_i)d\tau - \Pi \| \le \frac{1}{2}\alpha, i = 1, \ldots, N. \tag{15}$$

Then, the network (3) globally synchronizes.

CONTROL DESIGN

- Network science approach
- Structure assignment
- Optimization formulation
- Switching control

CONTROL DESIGN

Pinning Control

From Network Science – only control a small fraction of nodes (Li, Wang and Chen, 2004)

- Random pinning:
  Pin a fraction of randomly selected nodes
- Specific pinning:
  First pin the most important node, e.g. highest degree.
  Then select and pin the next important node.
  Continue $\cdots$ till control goal is achieved

Can exploit the network structure, e.g. hubs

But decentralized control on selected nodes

Structure assignment

- Controlled network

$$\dot{x}_i = f_i(x_i) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j + u_i, \quad i = 1, \ldots, N, \tag{16}$$

- Control action: re-set of the outer coupling

$$u_i = c\sum_{j=1}^{N} b_{ij}\Gamma x_j, \quad i = 1, \ldots, N, \tag{17}$$

where $B = (b_{ij})_{n\times n} \in \mathscr{B} \subset R^{n\times n}$ and $\mathscr{B}$ is a given control constraint set. The matrix $A + B$ for any matrix $B \in \mathscr{B}$ is symmetric and has zero row-sum.

CONTROL DESIGN

Structure assignment

Some typical forms of $\mathscr{B}$:

- Any $B \in \mathscr{B}$ is formed by adding or removing a certain number of links based on the existing links. The number can be pre-given.
- b_{ij} are obtained by adjusting the values of corresponding a_{ij}.
- Some boundedness on the entries of B, for example, $\sum_{j=1}^{N} | b_{ij} | \leq M_i$ for some pregiven constants $M_i > 0$.
- A combination of all above.

CONTROL DESIGN

Structure assignment

Definition. Let $(W_{n\times n}, R_{n\times m})$ be a matrix pair and $S \subseteq C^n$, $\mathscr{K} \subseteq R^{m\times n}$ be given sets. We say that the poles of the pair (W, R) can be assigned to the set S under the constraint set $\mathscr{K}$ if there exists $K \in \mathscr{K}$ such that the vector of eigenvalues of $W + RK$ belongs to S.

This notion is a generalization of pole assignment for linear systems when feedback is limited to an admissible set.

For simplicity, we only consider the case of equilibrium solution.

CONTROL DESIGN

Structure assignment

Let Q be the set of all $q \in R^n$ with the following property:

Property. There exist $n \times n$ matrix Π, which may be time-varying, uniformly positive definite matrices $P_i(t) \in \mathscr{P}\mathscr{C}^1_{n\times n}$ with $\| P_i \| \leq 1$ and constant $\alpha \geq 0$, all Π, P_i and α may be depending on q, such that

$$\dot{P}_i(t) + P_i(t)(\Pi + q_i\Gamma) + (\Pi + q_i\Gamma)^T P_i(t) \\ +\alpha I \prec 0,\ i = 1, \ldots, N,$$

$$\| \int_0^1 D f_i(s + \tau e_i) d\tau - \Pi \| \leq \frac{1}{2}\alpha, \quad i = 1, \ldots, N.$$

CONTROL DESIGN

Structure assignment

Theorem. Suppose $Q \neq \emptyset$. If the poles of the matrix pair (A, I) can be assigned to the set $\overline{Q} = \{\frac{1}{c}q \mid q \in Q\}$ under the constraint set $\mathscr{B}$, then , there exists $B \in \mathscr{B}$ such that the controllers $u_i = c\sum_{j=1}^{N} b_{ij}\Gamma x_j$ globally synchronize the network (16).

CONTROL DESIGN

Structure assignment

Theorem. Suppose there exist $q = (q_1, q_2, \cdots, q_N)^T$ with $q_1 = 0$, a unitary matrix $G = \{g_{ij}\}$, matrices Π_{ij}, $\Pi_{ij} = \Pi_{ji}$, $\delta_{ij} > 0$, $\delta_{ij} = \delta_{ji}$, $1 \le i, j \le N, i \ne j$, $\varepsilon_{ij} > 0$, $1 \le i < j \le N$, uniformly positive definite matrices $P_i(t) \in \mathscr{P}\mathscr{C}^1_{n\times n}$ with $\| P_i \| \le 1$ $\alpha_i \ge 0$, such that

$$\begin{aligned} &\dot{P}_i(t) + P_i(t)\Big(\sum_{j=1}^{N} g_{ji}^2 \int_0^1 Df_j(s+\tau e_j)d\tau + q_i\Gamma\Big) \\ + \quad &\Big(\sum_{j=1}^{N} g_{ji}^2 \int_0^1 Df_j(s+\tau e_j)d\tau + q_i\Gamma\Big)^T P_i(t) + \alpha_i I \prec 0, \end{aligned}$$

$$\sum_{j=1}^{N} | g_{jk} g_{ji} | \| \left(\int_0^1 Df_j(s+\tau e_j)d\tau - \Pi_{ik} \right) \| \le \delta_{ik},$$

CONTROL DESIGN

Structure assignment

$$
\begin{aligned}
2\sum_{k=2}^{N}\delta_{1k}\varepsilon_{1k}^{-1} &\leq \alpha_1, \\
2\sum_{k=i+1}^{N}\delta_{ik}\varepsilon_{ik}^{-1} + 2\sum_{l=1}^{i-1}\delta_{li}\varepsilon_{li} &\leq \alpha_i, 2 \leq i \leq N-1, \text{if } N \geq 3, \\
2\sum_{k=1}^{N-1}\delta_{kN}\varepsilon_{kN} &\leq \alpha_N.
\end{aligned}
$$

If $\frac{1}{c}G\,\mathrm{diag}\{q_1,\cdots,q_N\}G^T - A \in \mathscr{B}$, then the globally synchronization is achieved by the controller (17) with $B = \frac{1}{c}\,G\,\mathrm{diag}\{q_1,\cdots,q_N\}G^T - A$.

Optimization formulation

Dynamical network model:

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t), \quad i = 1, 2, \ldots, N, \qquad (18)$$

$A \in \mathbb{R}^{N\times N}$: 0-1 symmetric and irreducible

$$a_{ii} = -\sum_{\substack{j=1\\ j\neq i}}^{N} a_{ij} = -\sum_{\substack{j=1\\ j\neq i}}^{N} a_{ji}. \qquad (19)$$

Assumption. The equilibrium point $x_e = 0$ of the system

$$\dot{x}(t) = Df(s(t))x(t)$$

is exponentially stable.

CONTROL DESIGN

Optimization formulation

Graph theory

- An undirected graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ consists of $\mathscr{V} = \mathscr{V}(x_i)_{i=1}^N$ and $\mathscr{E} = \mathscr{E}(\overline{e_i})_{i=1}^M$;
- The incidence matrix $H = (h_1, h_2, \ldots, h_M) \in \mathbb{R}^{N \times M}$ is a matrix where $h_i \in \mathbb{R}^N$ with $h_{i_k} = 1$, $h_{i_l} = -1$ and all other entries 0 if the link $\overline{e_i} \in \mathscr{E}$ between nodes k and l;
- The Laplacian matrix L is the $N \times N$ matrix

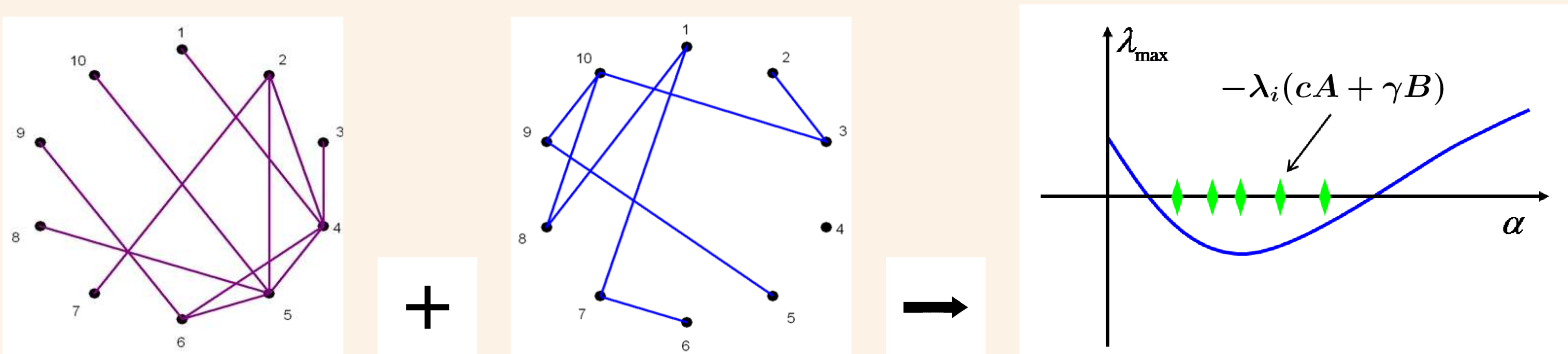
$$L = HH^\top = \sum_{i=1}^{M} h_i h_i^\top = -A; \qquad (20)$$

- The complement of $\mathscr{G}$ denoted by $\mathscr{G}^c$ consists of $\mathscr{V}$ and $\mathscr{E}^c = \mathscr{E}^c(\overline{e_i^c})_{i=1}^{M^c}$ with $M^c = \frac{N(N-1)}{2} - M$.

CONTROL DESIGN

Optimization formulation

Basic idea



Network controller:

$$\begin{cases} u_i = \gamma \sum_{j=1}^{N} b_{ij} \Gamma x_j, \\ \gamma \sum_{1 \le i < j \le N} b_{ij} \le \bar{d}, \end{cases} \tag{21}$$

CONTROL DESIGN

Switching control

Switching network controllers

$$u_i^{\sigma(t)} = \gamma_{\sigma(t)} \sum_{j=1}^{N} b_{ij}^{\sigma(t)} \Gamma x_j, \quad i = 1, 2, \ldots, N, \tag{22}$$

- $u_i^{\sigma(t)} \in \mathbb{R}^n$: the switching controller of node i;
- switching signal $\sigma(t) : [0, \infty) \rightarrow \mathscr{M} = \{1, \ldots, m\}$
- $\gamma_k > 0$: the control gain of u_k;
- $B_k = (b_{ij}^k)_(N \times N)$: the outer coupling matrix of u_k.

γ_k and B_k satisfy the energy constraint (23) with $\bar{d} > 0$,

$$\gamma_k \sum_{1 \leq i < j \leq N} b_{ij}^k \leq \bar{d}. \tag{23}$$

CONTROL DESIGN

Switching control

Theorem. Consider the unbounded sync region $S = [\alpha_1, \infty)$. For a given candidate controller set $\mathscr{U}$, if the solution λ_2^* of the convex optimization (24) satisfies $\lambda_2^* \leq -\alpha_1$, then the synchronization of the network (18) is achieved under the switching law (25).

$$\begin{aligned} \min \quad & \lambda_2(cA + \sum_{k=1}^{m} \theta_k \gamma_k B_k) \\ s.t. \quad & \sum_{k=1}^{m} \theta_k = 1 \\ & \theta_k \in [0,1], \;\; k = 1, 2, \ldots, m \end{aligned} \tag{24}$$

CONTROL DESIGN

Switching control

Switching law

$$\sigma(t) = k, \text{ if } (t,e) \in \Omega_k, \tag{25}$$

where

$$\begin{aligned}\Omega_k = \{(t,e)|e^\top(\dot{P} + (I_N \otimes Df(s(t)) + A_k \otimes \Gamma)^\top P \\ +P(I_N \otimes Df(s(t)) + A_k \otimes \Gamma))e < 0\}\end{aligned} \tag{26}$$

and

$$\dot{P}_i + (Df(s(t)) + \lambda_i \Gamma)^\top P_i + P_i(Df(s(t)) + \lambda_i \Gamma) < 0$$

$P = (\Phi \otimes I_n)\bar{P}(\Phi^\top \otimes I_n)$ and $\bar{P} = \text{diag}\{P_1, P_2, \ldots, P_N\}$

REFERENCES

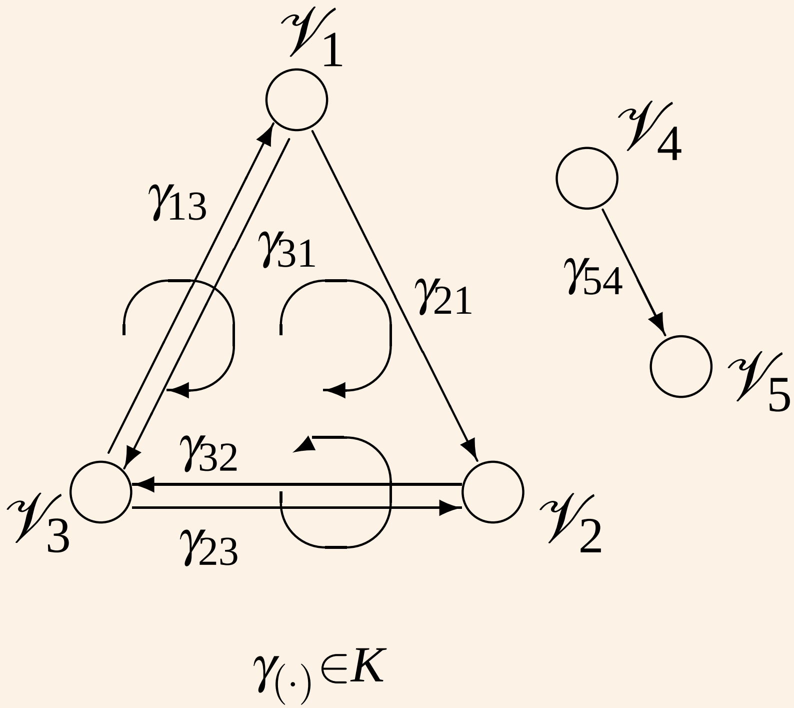
- D.J.Hill and J.Zhao, "Global synchronization of complex dynamical networks with non-identical nodes," Proc 47th IEEE Conference on Decision and Control, Cancun, Mexico, pp.817-822, December 2008.
- J.Zhao and D.J.Hill, "Global bounded synchronization of general dynamical networks with non-identical nodes," submitted to IEEE TAC.
- J.Zhao, D.J.Hill and T.Liu, "Synchronization of dynamical networks with non-identical nodes: criteria and control," to be submitted.
- T.Liu, D.J.Hill and J.Zhao, "Synchronization of dynamical networks by network control," Proc 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference, Shanghai, China, December 2009, pp. 1684-1689.

OTHER WORK AND IDEAS

- Small-gain theory (TFLiu, Hill and Jiang, CDC 2009)
- Passivity approach (Arcak, IEEE TAC 2007)
- Time-delays (TLiu, Hill and Zhao, submitted to NOLCOS)
- Switched networks (Zhao and Hill, Automatica, 2009)
- Impulsive network control (BLiu and Hill, CDC 2008)
- Mean-field control (Caines, CDC 2009)
- Tractability (Swigart and Lall, CDC 2009)

Small-gain theory

Theorem (Lyapunov-ISS cyclic-small-gain) The dynamical network is ISS, if the composition of the Lyapunov-ISS gains along every cycle is less than Id.



$$\gamma_{13} \circ \gamma_{31} < \text{Id}$$
$$\gamma_{23} \circ \gamma_{32} < \text{Id}$$
$$\gamma_{12} \circ \gamma_{23} \circ \gamma_{31} < \text{Id}$$

Ref: T.Liu, D.J.Hill and Z-P.Jiang, CDC 2009

CONCLUSIONS

- Feedback networks with non-id nodes model engineering systems
- Stability theory
- Control synthesis and design
- Scaling to large systems - bring in computer scientists?

Thank you