Long-run Negotiations with Dynamic Accumulation

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The Dynamic Problem

- Two parties can share a surplus between investment and consumption: how much do they invest? How do they split the residual surplus among themselves?
- *Dynamic accumulation: the level of investment affects the future capital stock and consequently, the surplus available in the following bargaining stage*.
- Examples: partners in a business; trade talks, negotiations on climate change.

Literature

- Two major strands:
- **Hold-up problem** (e.g., Gibbons (1992), Muthoo (1996), Gul (2001)).

  Typically, only one party is involved in the investment problem, moreover, the investment is once for all.
- Differently, we look at problem where parties *jointly* and *repeatedly* need to decide how much to invest and consume.

Literature

- **Tragedy of the commons** (e.g., Levhari and Mirman (1980), Dutta and Sandaram (1993)) Common-property resource games.
- The typical framework does not include any negotiations.
- Exceptions: Houba et al. (2000) and Sorger (2006), which introduce bargaining in a simplified manner.

Literature

- Muthoo (1999, sec. 10.3): first to consider a repeated (non-cooperative) bargaining model with investment decisions in addition to the standard consumption decisions.
- Focus is on *steady-state SPE*. Infinite number of surpluses of the same size (Muthoo, 1995).
- Given the 'simple' investment problem, parties can be risk neutral.

Outline

- Model
- Types of equilibria
- Main incentives in dynamic frameworks
- Efficiency in bargaining
- Some robustness

The Model

- Alternating stages: Production and Bargaining
- Time is discrete, $t = 0,1,2, \ldots$
- *Production*: surplus is given by $F(k_t) = Gk_t$, given k_0 and $G > 0$. Production takes time (τ).
- *Bargaining*: Two players: 1 and 2. Alternating-offer procedure á la Rubinstein (1982).

A proposal by Player i is a pair $({}_ix_t, {}_iI_t)$:

${}_iI_t$ = investment level,

${}_ix_t$ = share demanded by *i* over the residual surplus.

After an acceptance

- If the proposal is accepted, the bargaining stage ends, per-period utilities:

$$ {}_ic_t^{1-\eta}/(1-\eta) \qquad \text{for } \eta \neq 1 $$

$$ \ln {}_ic_t \qquad \text{for } \eta = 1 $$

where ${}_ic_t=(F(k_t)-{}_iI_t){}_ix_t$ and ${}_jc_t=(F(k_t)-{}_iI_t)(1-{}_ix_t)$.

- Output at t+1 is $F(k_{t+1})$, with $k_{t+1}={}_iI_t+(1-\lambda)k_t$, where λ is the depreciation rate ($0 \leq \lambda \leq 1$).

After a rejection

- If the proposal is rejected a time period, Δ passes accepted.
- Discount factors:

$$\delta_i = \exp(-h_i\Delta)$$

$$\alpha_i = \exp(-h_i \tau)$$

where h_i is player i's rate of time preference.

Example of a possible time line

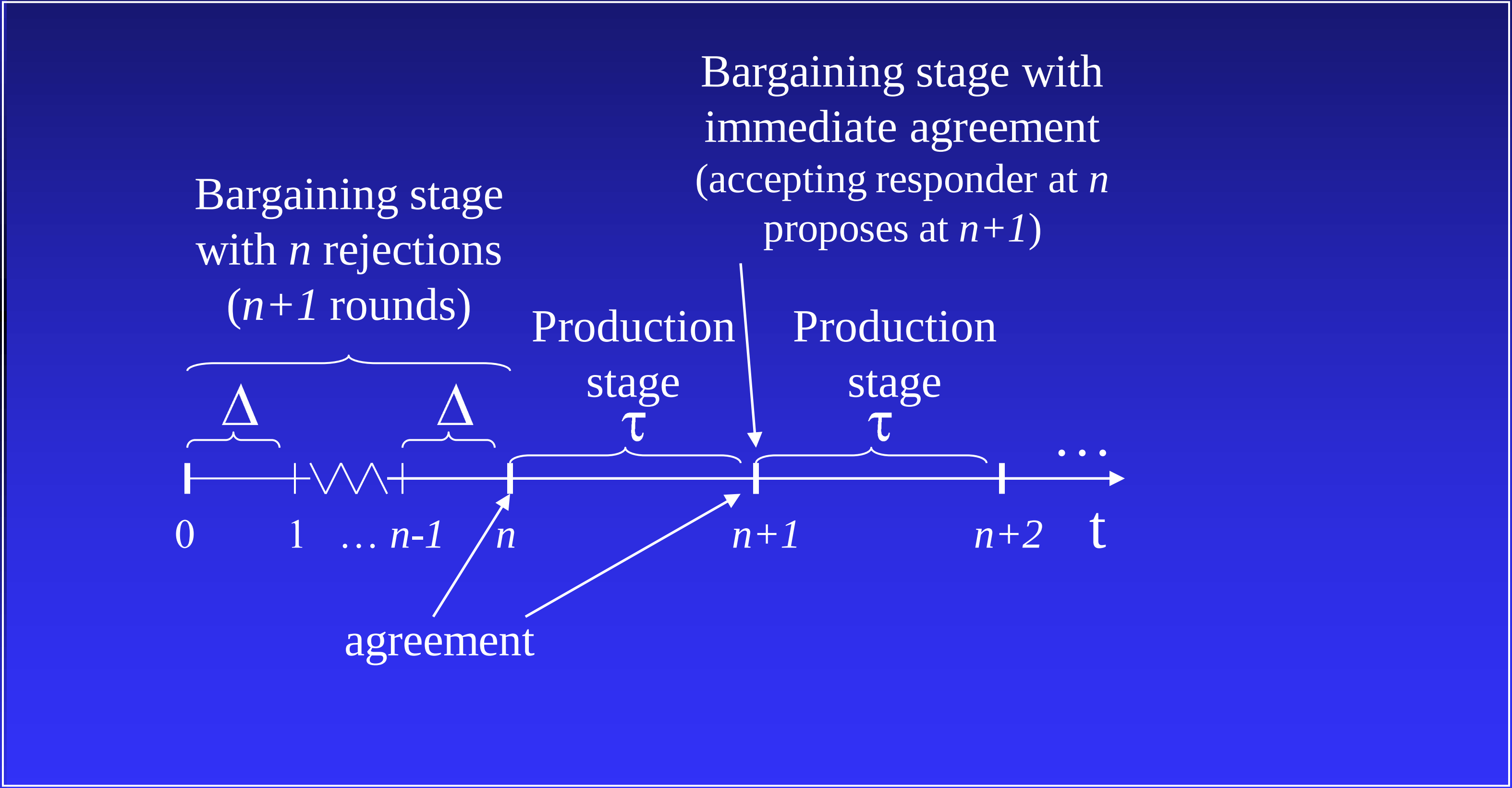

Figure 1. Time line for a game with n (0) rejections in the first (second, respectively) bargaining stage.

Equilibrium

- Stationary *Markov Subgame Perfect Equilibra* (MPE).
- State variable: k_t.
- Natural candidate: linear strategies: x_i and φ_i $=_i I_t/k_t$ are constant. Why?
- Asymptotic approach (number of bargaining stages is finite but tends to infinity).

Results: Characterisation of the MPE

- There is a unique MPE with immediate agreement
- But three possible types of MPE:

  At least one player consumes all the residual surplus, $x_i = 1$ (Ultimatum-like MPE).

  B*oth* demands are less than 1.
- In a frictionless bargaining game, symmetric players behave efficiently.
- Typically, they either under-invest (for $\eta < 1$) or over-invest (for $\eta > 1$).

Results: Effects in a Dynamic Framework

- The more patient party consumes *less* than his opponent, if production is sufficiently long.
- The more patient a party is the higher the investment plan of *all* parties.
- Patience can make the rival better off.
- Note on log utility, generally the MPE strategies are time-dependent. Time-invariant rules can be derived only at the steady-state or at the limit for $\Delta \to 0$.

The Recursive Problem

(1) $$V_i(k_t) = \max_{\substack{0 \le x_i \le 1 \\ -(1-\lambda) \le \varphi_i \le G}} \frac{[x_i(G-\varphi_i)k_t]^{1-\eta}}{1-\eta} + \alpha_i W_i(k_{t+1})$$

s.t.

(2) $$W_j(k_t) = \frac{[(1-x_i)(G-\varphi_i)k_t]^{1-\eta}}{1-\eta} + \alpha_j V_j(k_{t+1}) \ge \delta_j V_j(k_t)$$

where

(3) $k_{t+1} = k_t(1-\lambda+\varphi_i)$ in case of acceptance, for i $\neq$ j, and i, j = 1,2.

Guesses

$$V_i(k_t) = \phi_{\mathrm{i}} \frac{k_t^{1-\eta}}{1-\eta}$$

$$W_i(k_t) = \mu_i \frac{k_t^{1-\eta}}{1-\eta}$$

Ultimatum-like MPE

- Focus on $\eta<1$. Let l = G+1-λ

Result 1. For $\eta>1/2$, if $\delta \le (\alpha l^{1-\eta})^{1/(2\eta-1)}<1$, there is a unique MPE in which the proposers consume all the residual surplus and

$$\varphi_i + (1-\lambda) = \left(l\alpha^2 \right)^{\frac{1}{2\eta-1}}$$

e.g., for $\eta=2/3$, $\delta_i= 0.9$, $\alpha_i = 0.8$, then $l\in[1.76,1.95)$

Ultimatum-like MPE

- When η is sufficiently high, parties prioritise investment.
- The more patient a party is, the higher the investment plan of *all* parties.

$$\varphi_i + (1-\lambda) = \quad l(\alpha_i^{\eta}\alpha_j^{1-\eta})^{2}\ {}^{\frac{1}{2\eta-1}}$$

- A more patient party makes the opponent better off (μ_j and ϕ_j increase with α_i).

Asymmetric Ultimatum-like MPE

Result 2. If player *i* is sufficiently more patient than *j*, then only player *i* can demand an extreme share (x_i=1).

Interior MPE

Result 3. There is a unique MPE:

$$x_i = 1/(1 + m_i^{1/\eta})$$

$$\varphi_i = G - l(1 + m_i^{1/\eta})/\psi_i$$

where $(m_i, \psi_i) \in M_i$ and solves a highly non-linear system.

Properties in a simple case

- Symmetry (h_i=h) and $\eta = ½$.

Result 4. For $\eta = 1/2$ and h_i=h, with i=1,2, if $\alpha^2 l < 1$, there is a unique symmetric MPE, players under-invest, unless $\Delta \rightarrow 0$.

Result 5. The MPE demand x is decreasing with δ and increasing with α, while the SPE investment φ is increasing in both δ and α.

Properties in a simple case

Result 6. The more patient parties are, the higher the investment plan. However, the cost of a higher investment when *only* δ increases is paid mainly by the proposer (the responder increases his consumption level).

Properties for asymmetric parties

Result 7. For $\eta < 1$,

Investment: the more patient party invests shares larger than his opponent's. The more patient a party becomes, the higher the investment plans of all parties. (imp. of $\alpha_i - \alpha_j$)

Consumption: the more patient party consumes more than his opponent, unless l is sufficiently large and production is sufficiently long.

Asymmetries

i \ j	$\alpha_j = 0.3$ $\delta_j = 0.4$	$\alpha_j = 0.4$ $\delta_j = 0.5$	$\alpha_j = 0.4$ $\delta_j = 0.6$	$\alpha_j = 0.6$ $\delta_j = 0.7$	$\alpha_j = 0.8$ $\delta_j = 0.9$
$\alpha_i = 0.3$ $\delta_i = 0.4$	0.883, 0.020	0.823, 0.032 0.897, 0.034	0.727, 0.039 0.912, 0.041	0.664, 0.082 0.931, 0.097	0.389, 0.221 0.971, 0.255
$\alpha_i = 0.4$ $\delta_i = 0.5$		0.842, 0.046	0.750, 0.052 0.862, 0.052	0.688, 0.096 0.891, 0.107	0.404, 0.227 0.953, 0.259
$\alpha_i = 0.4$ $\delta_i = 0.6$			0.778, 0.058	0.717, 0.097 0.813, 0.108	0.430, 0.223 0.907, 0.251
$\alpha_i = 0.6$ $\delta_i = 0.7$				0.760, 0.145	0.461, 0.249 0.883, 0.267
$\alpha_i = 0.8$ $\delta_i = 0.9$					0.633, 0.305

Table 1. For $\eta = 1/2$ and l=0.7, MPE proposals first for *i* (x_i,r_i), then for *j*, with $r_i=1-\lambda+\varphi_i$

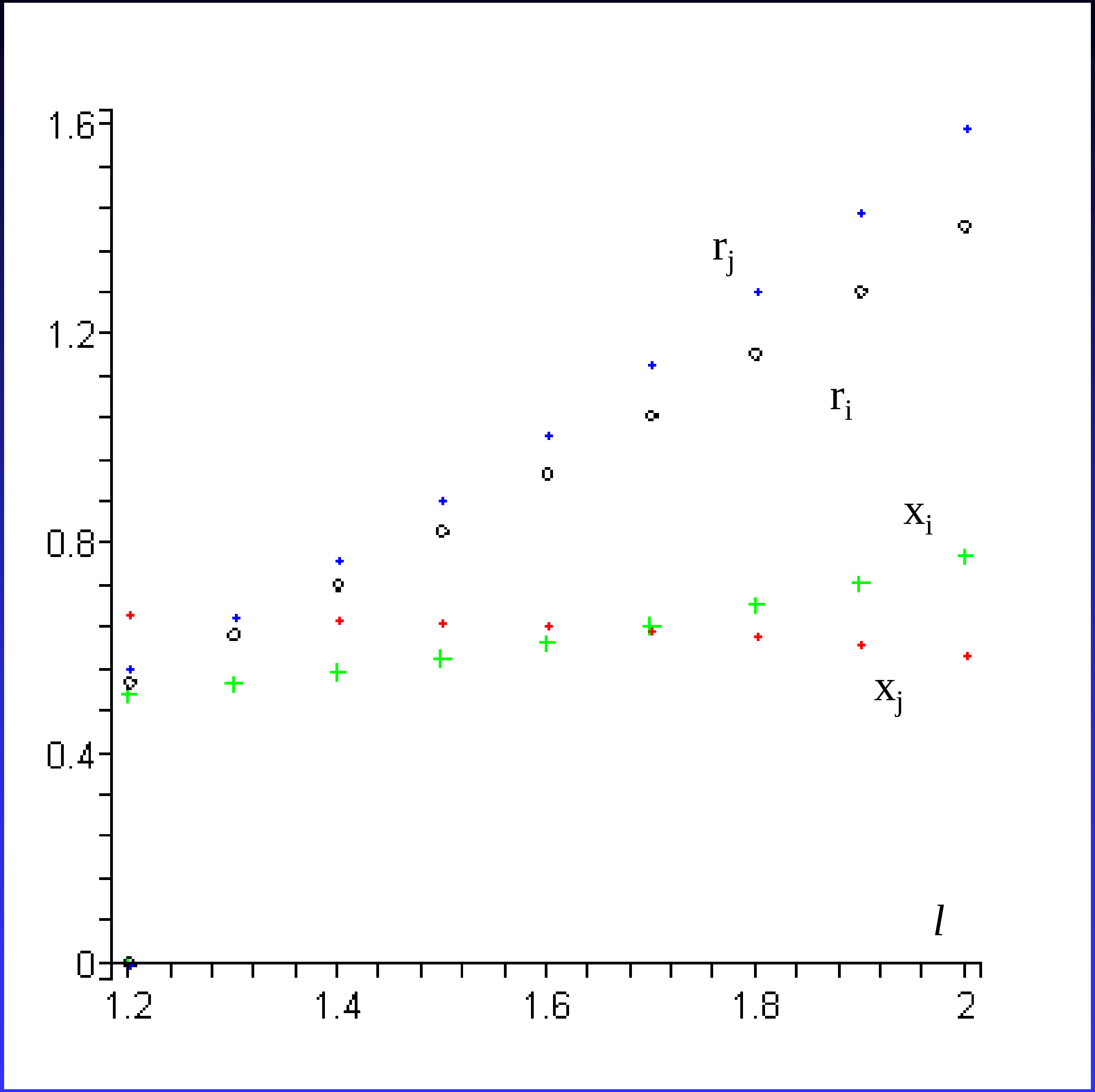

Fig. 2 MPE strategies for $\eta = 1/2$, $\alpha_i = 0.5$, $\alpha_j = 0.7$, $\delta_i = 0.9$, and $\delta_j = 0.95$.

Properties for asymmetric parties

Result 8. For $\eta > 1$, players consume less than half of the residual surplus (unless there are strong asymmetries).
The most patient player invests *less* than his rival and demands to consume a larger share (unless production is long and l is sufficiently small).

i \ j	$\alpha_j = 0.3$ $\delta_j = 0.4$	$\alpha_j = 0.4$ $\delta_j = 0.5$	$\alpha_j = 0.8$ $\delta_j = 0.9$	$\alpha_j = 0.9$ $\delta_j = 0.99$
$\alpha_i = 0.3$ $\delta_i = 0.4$	0.113, 1.043	0.119, 1.077 0.129, 1.041	0.087, 1.143 0.461, 1.051	0.014, 1.166 0.911, 1.151
$\alpha_i = 0.4$ $\delta_i = 0.5$		0.136, 1.075	0.100, 1.142 0.458, 1.073	0.018, 1.165 0.908, 1.152
$\alpha_i = 0.8$ $\delta_i = 0.9$			0.345, 1.128	0.087, 1.161 0.842, 1.156
$\alpha_i = 0.9$ $\delta_i = 0.99$				0.480, 1.162

Table 2. For $\eta = 2$, $l = 1.5$ equilibrium as described in table 1.

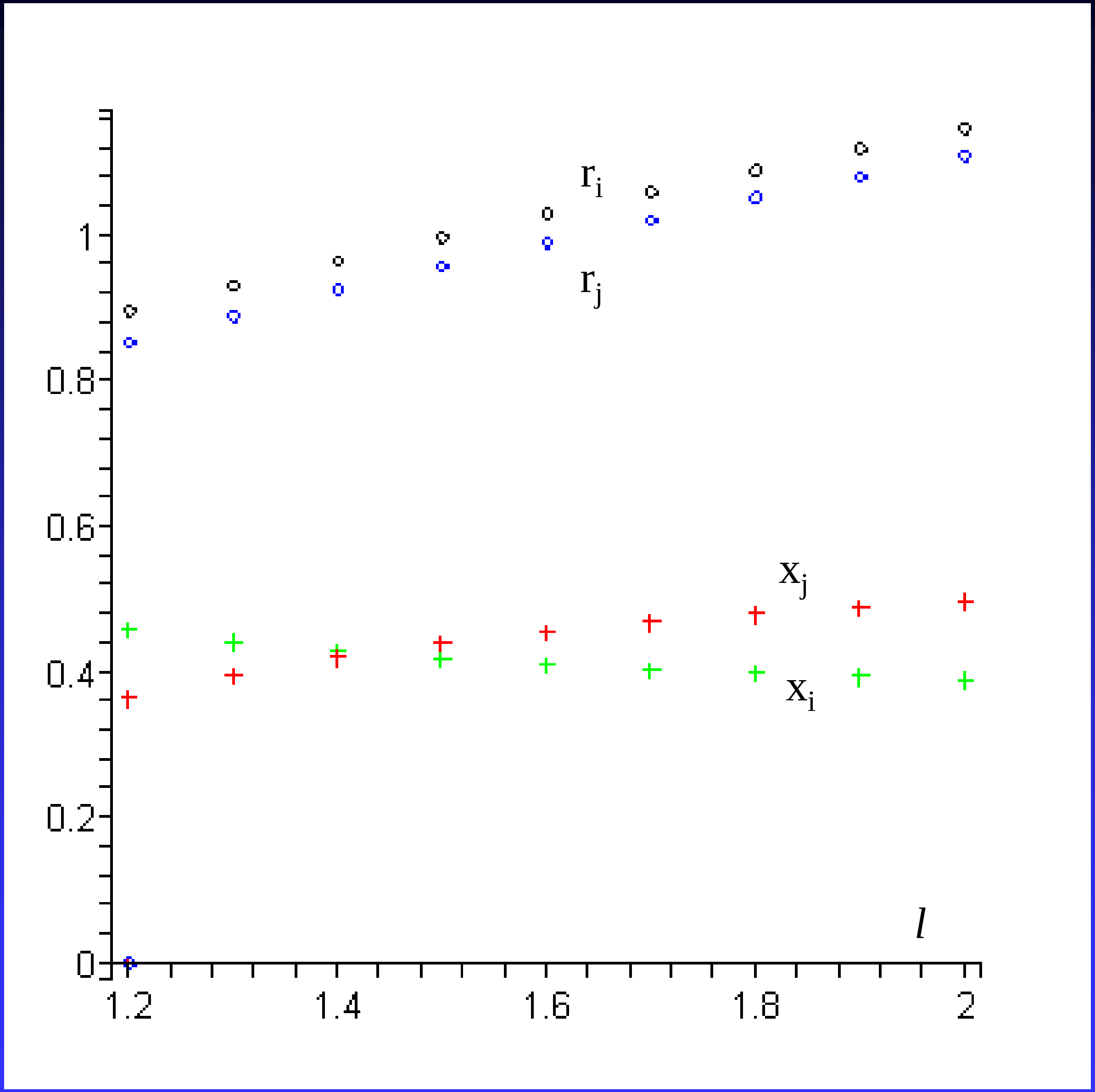

Fig. 3 MPE strategies for $\eta = 2$, $\alpha_i = 0.5$, $\alpha_j = 0.7$, $\delta_i = 0.9$, and $\delta_j = 0.95$.

Efficiency

- Frictionless bargaining is efficient.
- For $\eta > 1$, impatient parties over-invest.
- Example: For $\delta = 0.99$, $\alpha = 0.9$, $\eta = 2$, $l = 1.5$

  $\varphi_i^E = 1.162 < 1.183 = \varphi_i$ investment when $\delta = 0.93$, ceteris paribus).

Patience can be weakness

- Assume that player *j* is more patient than *i*, production is sufficiently long and η is sufficiently large, then

Result 9. Patience can make a rival better off.

	$\eta = 1/2$ $l = 1.5$	$\eta = 2/3$ $l = 1.6$	$\eta = 2$ $l = 1.2$	$\eta = 3$ $l = 1.3$
$\alpha_i = 0.63$ $\delta_i = 0.9$ $\alpha_j = 0.8$ $\delta_j = 0.95$	0.783, 1.128 (1.365, 1.228) 0.720, 1.258 (2.828, 2.687)	0.832, 1.131 (1.795, 1.616) 0.818, 1.320 (3.680, 3.500)	0.415, 0.974 (25.677, 23.109) 0.344, 0.933 (58.745, 55.808)	0.452, 1.004 (128.120, 115.307) 0.438, 0.983 (243.329, 231.163)
$\alpha = 0.8$ $\delta = 0.95$	0.965, 1.343 (3.263, 3.100)	0.942, 1.369 (3.852, 3.660)	0.380, 0.991 (54.048, 51.345)	0.450, 1.018 (233.216, 221.556)

Table 3. MPE proposal (ϕ and μ), related to player i and j respectively for asymmetric cases.

Random-proposer procedure

- Results above are robust.

Result 10. For $\eta < 1$, if player *i* is more impatient than player *j* and the probability of proposing for player *i* increases then player *j*′s level of investment decreases while player *i*′s increases (vice-versa for $\eta > 1$)

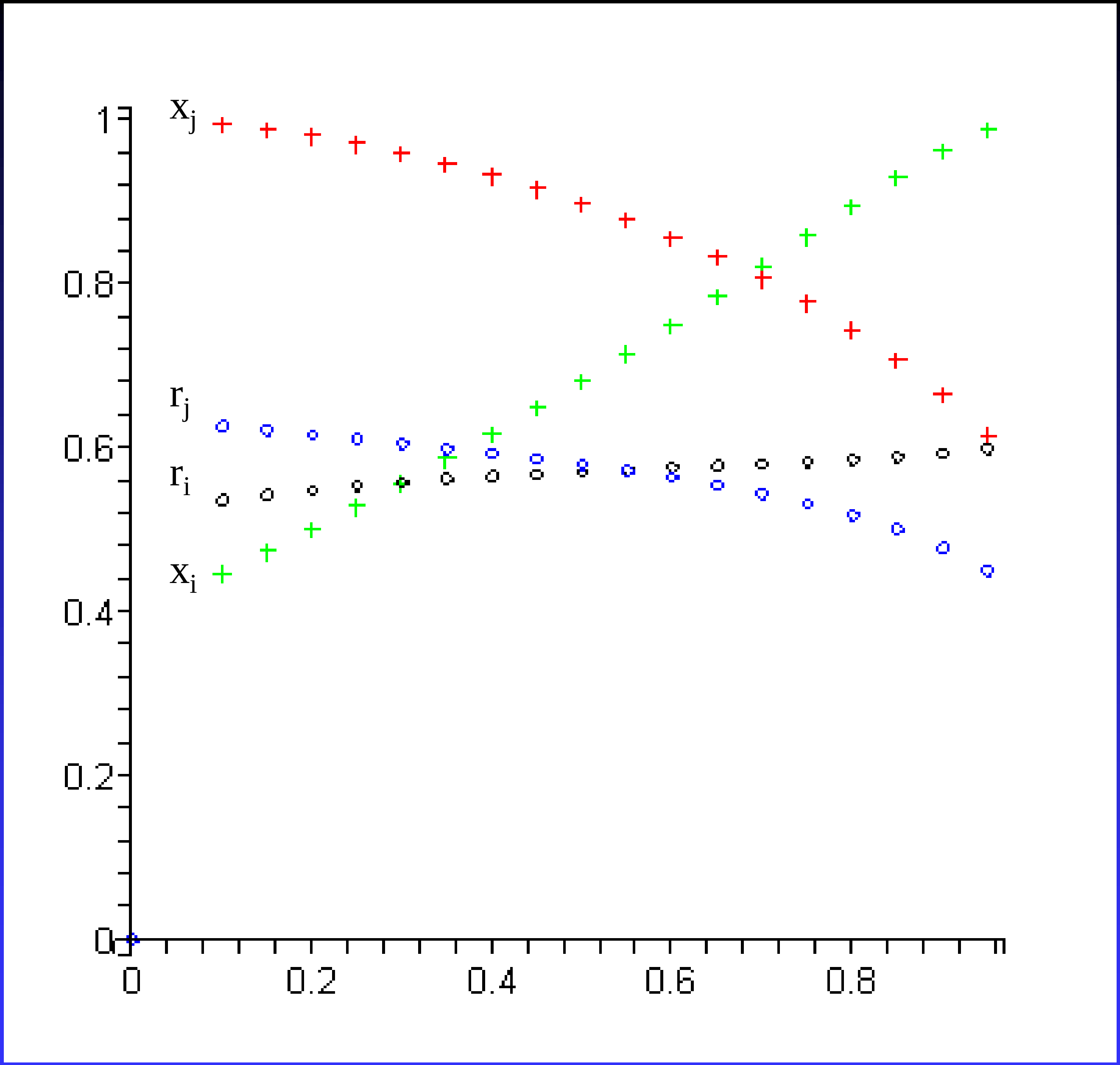

Fig.4. MPE, $\eta = 1/2$, $l = 1$, $\alpha_i = 0.78$, $\alpha_j = 0.8$, $\delta_i = 0.82$ and $\delta_j = 0.9$

Conclusion

- Extreme demands are possible in a dynamic framework.
- The most patient party demands a larger share of the residual surplus, unless production is sufficiently long.
- Moreover, he will invests more only if $\eta<1$.
- Bargaining is efficient only in a frictionless world, otherwise parties may either over- or under-invest.
- Patience can be weakness.