# Price competition and robustness to strategic uncertainty

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or...

A THEORETICAL IDEA THAT

happens to

FIT SOME EXPERIMENTAL EVIDENCE

An alternative topic for this workshop could have been:

# ROBUST SETS IN EVOLUTIONARY GAME DYNAMICS

- Benaïm M. and J. Weibull (2003): "Deterministic approximation of stochastic evolution in games", *Econometrica* 71, 873-903.

- Ritzberger K. and J. Weibull (1995): "Evolutionary selection in normal-form games", *Econometrica* 63, 1371-1399.

- the word "robust" -

# 1 Introduction

We analyze a class of price competition games with

1. continuum strategy sets

2. discontinuous profit functions

3. continuum of price equilibria

[Dixon (1990), Dastidar (1995), Vives (1999), Chowdhury and Sengupta (2004), Weibull (2006)]

- Even the slightest uncertainty about competitors' price choices might lead firms to deviate

- Reasonable to require equilibria to be robust to small amounts of uncertainty about other players' strategies

  [Selten (1975), Simon and Stinchcombe (1995), Al-Najjar (1995), Carlsson and Ganslandt (1998)]

- Recent evidence from laboratory experiments

  [Abbink and Brandts (2008), Argenton and Müller (2009)]

## 2 Robustness against strategic uncertainty

- Let $G = (N, S, \pi)$ be an $n$-player normal-form game with:

  - player set $N = \{1, ..., n\}$

  - pure-strategy set of each player: $S_i = \mathbb{R}$

  - payoff functions $\pi_i : S \to \mathbb{R}$

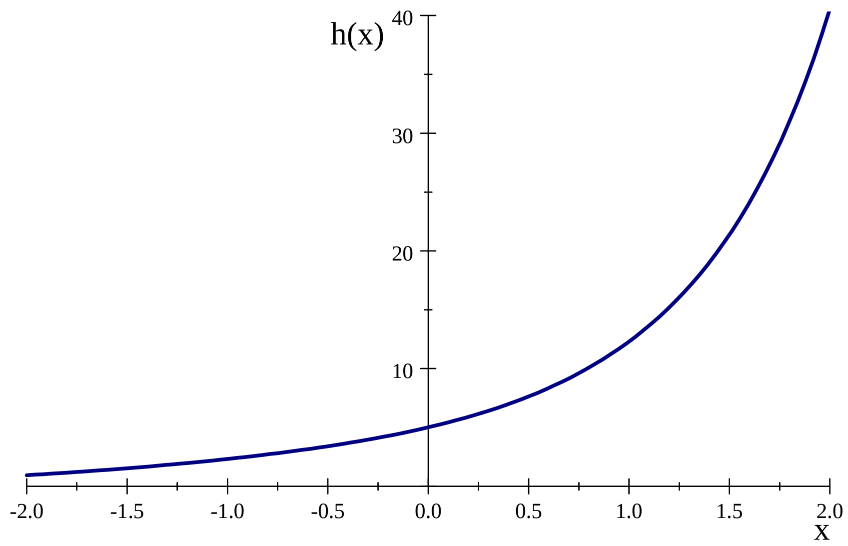
- Let $\mathcal{F}$ be the class of c.d.f:s $F : \mathbb{R} \to [0, 1]$ with:

  - everywhere positive and continuous density $f = F'$

  - non-decreasing hazard rate

$$h(x) = \frac{f(x)}{1 - F(x)}$$

- Examples are the normal, exponential and Gumbel distributions (sufficient that $f$ be log-concave):



**Definition 2.1** *Given $t \geq 0$, a strategy profile $\mathrm{s}$ is a* **t-equilibrium** *if, for each player $i$, the strategy $s_i$ maximizes $i$'s expected payoff under the probabilistic belief*

$$\tilde{s}_{ij} = s_j + t \cdot \varepsilon_{ij} \quad \forall j \neq i$$

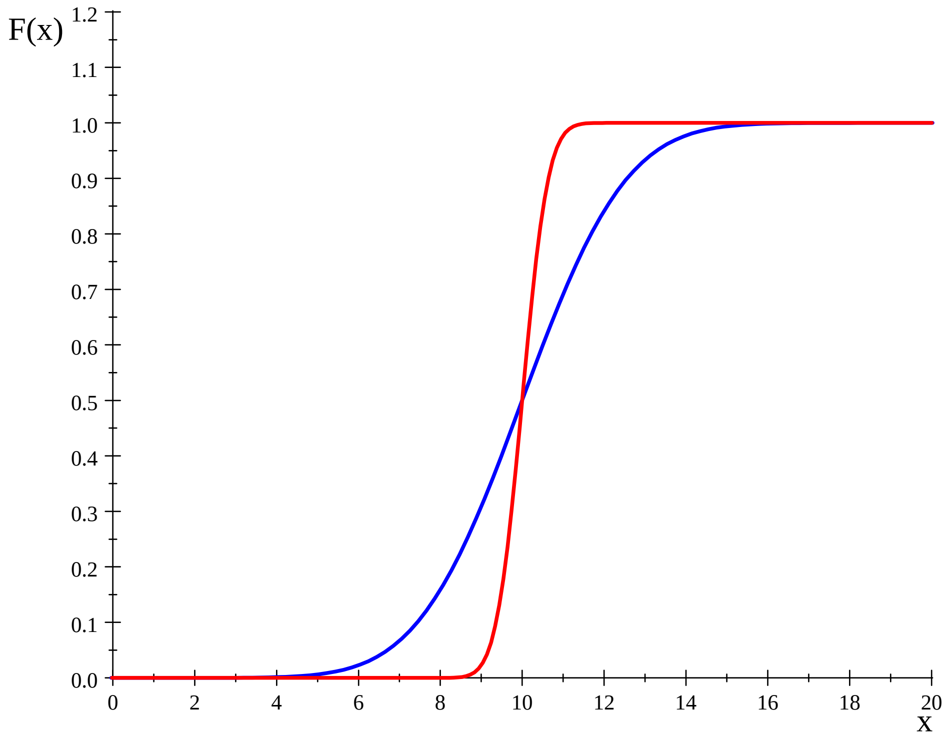
*for independent "noise" terms $\varepsilon_{ij} \sim \Phi_{ij} \in \mathcal{F}$*

**Remark 2.1** *For $t = 0$: $\Leftrightarrow$ Nash equilibrium*

**Remark 2.2** *For $t > 0$: $\tilde{s}_{ij} \sim F^t_{ij} \in \mathcal{F}$ where*

$$F^t_{ij}(x) = \Phi_{ij}\left(\frac{x - s_j}{t}\right) \quad \forall x \in \mathbb{R}$$

**Example 2.1** *For* $s_j = 10$ *and* $\Phi_{ij}$ *normal:*



**Remark 2.3** *Let $t > 0$ and $\Phi_{ij} \in \mathcal{F}$ $\forall i \in N, j \neq i$. A strategy profile $\mathrm{s}$ is a $t$-equilibrium of $G = (N, S, \pi)$, with $\varepsilon_{ij} \sim \Phi_{ij}$, if and only if it is a NE of $G^t = \left(N, S, \pi^t\right)$, where*

$$\pi_i^t(\mathbf{s}) = \mathbb{E}_{\mathbf{\Phi}_{ij}}\left[\pi_i\left(s_i, \tilde{s}_{-i}\right)\right]$$

**Definition 2.2** *A Nash equilibrium $\mathrm{s}^*$ of $G$ is* **robust to strategic uncertainty** *if $\exists$ c.d.f:s $\left\{\Phi_{ij} \in \mathcal{F} : \forall i, j \neq i\right\}$ and a sequence of $t$-equilibria such that $s^t \rightarrow s^*$ as $t \rightarrow 0$.*

$\mathrm{s}^*$ *is* **strictly** *robust if this holds for* **all** *collections $\left\{\Phi_{ij} \in \mathcal{F} : \forall i, j \neq i\right\}$.*

**Example 2.2** *Classical Bertrand duopoly with linear demand and constant unit cost:*

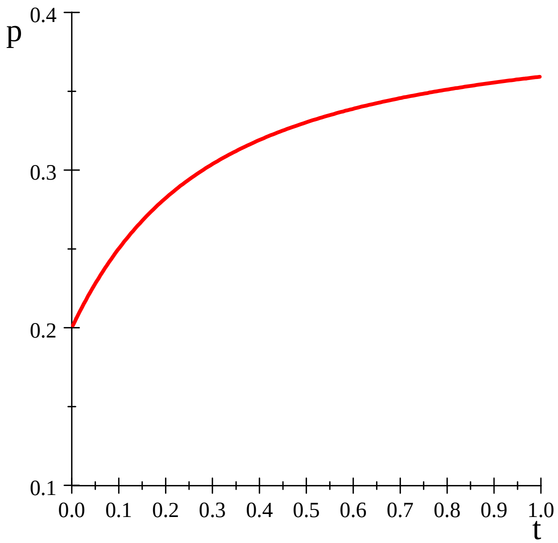
$$\Pi^m(p) = (1-p)(p-c)$$

*Uniqe NE:*$(c, c)$*. However, weakly dominated.*

*A necessary FOC for* $(p, p)$ *to be a* $t$*-equilibrium:*

$$t \cdot \frac{\Pi'(p)}{\Pi(p)} = h(0)$$

For $c = 0.2$:



# 3 Price competition with convex costs

- $n \geq 2$ identical firms
- market for a homogeneous good
- cost function $C$ with $C', C'' > 0$
- demand function $D$ with $D' < 0$
- all firms simultaneously set their prices $p_i$
- let $\mathbf{p} = (p_1, p_2, ..., p_n)$

- the market price: $p_0 = \min_i p_i$

- let $k = |\{i : p_i = p_0\}|$.

- firm $i$ faces demand

$$D_i(\mathbf{p}) = \begin{cases} D(p_0)/k & \text{if } p_i = p_0 \\ 0 & \text{otherwise} \end{cases}$$

- each firm is required to serve all its clients

- profit to firm $i$:

$$\pi_i(\mathbf{p}) = \begin{cases} p_0 D(p_0)/k - C\left[D(p_0)/k\right] & \text{if } p_i = p_0 \\ 0 & \text{otherwise} \end{cases}$$

- for $k = 1, 2, ..., n$ let

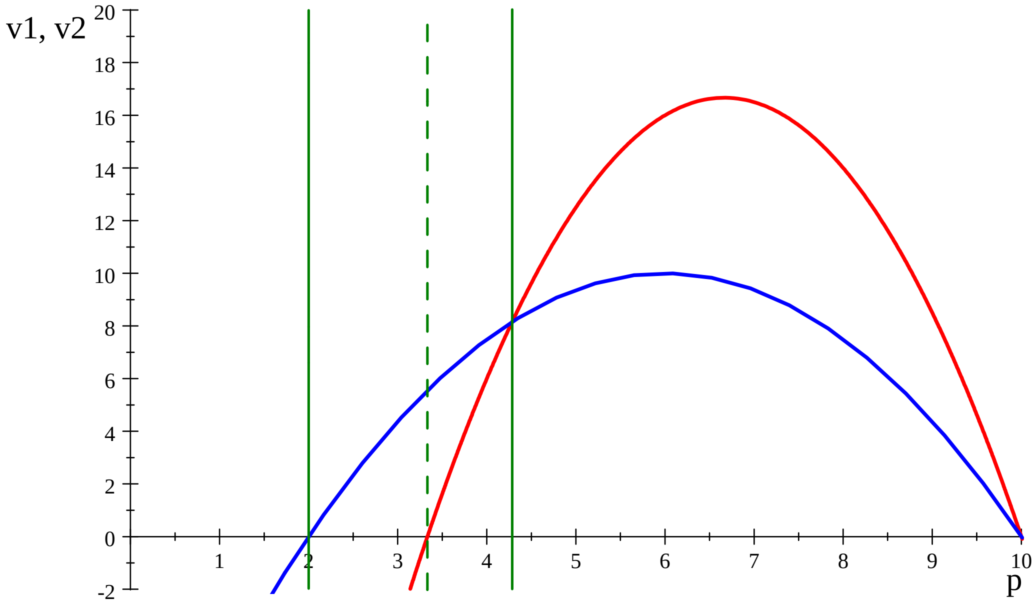
$$v_k(p) = pD(p)/k - C[D(p)/k]$$

1. Such a game $G$ has a continuum of Nash equilibria, all symmetric

2. $P^{NE} = [\check{p}, \hat{p}]$

- where $v_n(\check{p}) = 0$ and $v_n(\hat{p}) = v_1(\hat{p})$

- $\exists!$ price $\bar{p} \in (\check{p}, \hat{p})$ with $v_1(\bar{p}) = 0$



v1, v2
20
18
16
14
12
10
8
6
4
2
0
-2
1
2
3
4
5
6
7
8
9
10
p

# 4 Robust price equilibrium

- Perturbed game $G^t = \left(N, S, \pi^t\right)$

**Proposition 4.1** *The price profile* $(\bar{p}, ..., \bar{p})$ *is strictly robust to strategic uncertainty. No other strategy profile is robust to stategic uncertainty.*

**Proof:** A number of technical observations centered around the equation system

$$t \cdot v_1' (p_i) = v_1 (p_i) \sum_{j \neq i} h_{ij} \left( \frac{p_i - p_j}{t} \right) \quad \forall i, j \neq i$$

**Intuition:** Asymmetric incentive to deviate for higher prices and also for lower prices

# 5 Example

- duopoly

- identical firms with quadratic cost functions

- linear demand

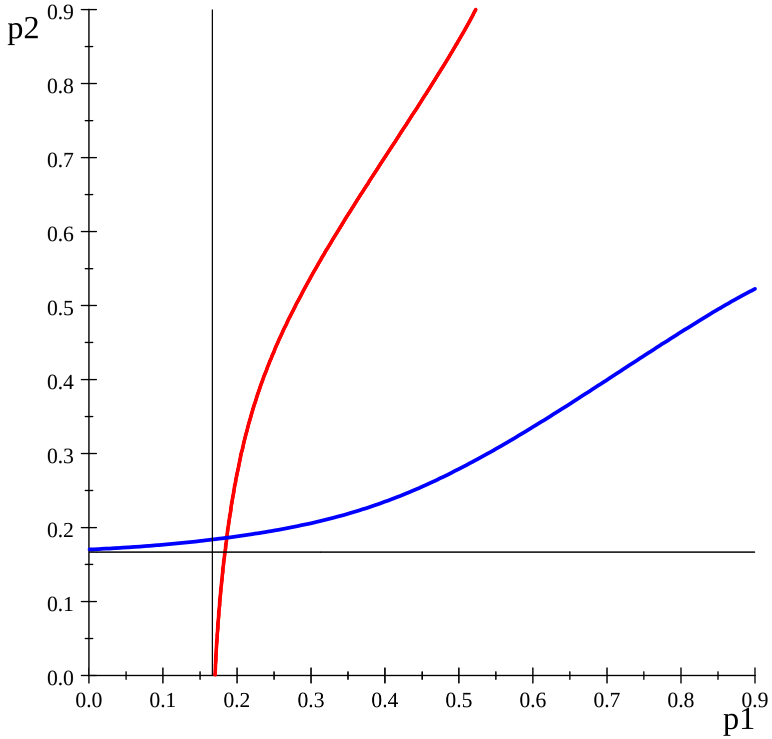
- normally distributed noise

$$\check{p} \approx 0.091$$

$$\bar{p} \approx 0.167$$

$$\hat{p} \approx 0.231$$

$$p^{mon} \approx 0.583$$



# 6 Conclusion

- Empirical support: Abbink K. and J. Brandts (2008)

- Application to other games: The Nash demand game (Nash 1950)

- Clarify connections with other refinements!